Question	Scheme	Marks	AOs
8(a) Way 1	$\left(\frac{1}{\operatorname{cosec}\theta-1}+\frac{1}{\operatorname{cosec}\theta+1}\equiv\right)\frac{\operatorname{cosec}\theta+1+\operatorname{cosec}\theta-1}{(\operatorname{cosec}\theta-1)(\operatorname{cosec}\theta+1)}$	B1	1.1b
	$\equiv\frac{2\operatorname{cosec}\theta}{\operatorname{cosec}^2\theta-1}\equiv\frac{2\operatorname{cosec}\theta}{\cot^2\theta}$ or e.g. $\equiv\frac{2\sin\theta}{1-\sin^2\theta}=\frac{2\sin\theta}{\cos^2\theta}$	M1	1.1b
	$\frac{2\operatorname{cosec}\theta}{\cot^2\theta}\equiv\frac{2}{\sin\theta}\times\frac{\sin^2\theta}{\cos^2\theta}\equiv 2\tan\theta\sec\theta$* or $\frac{2\sin\theta}{\cos^2\theta}\equiv 2\tan\theta\sec\theta$*	A1*	2.1
		(3)	
(a) Way 2	**"Meets in the middle"**		
	$\left(\text{LHS}=\frac{1}{\operatorname{cosec}\theta-1}+\frac{1}{\operatorname{cosec}\theta+1}\equiv\right)\frac{\operatorname{cosec}\theta+1+\operatorname{cosec}\theta-1}{(\operatorname{cosec}\theta-1)(\operatorname{cosec}\theta+1)}$	B1	1.1b
	$\equiv\frac{2\operatorname{cosec}\theta}{\operatorname{cosec}^2\theta-1}\equiv\frac{2\operatorname{cosec}\theta}{\cot^2\theta}$	M1	1.1b
	$\text{RHS}=2\tan\theta\sec\theta\equiv\frac{2\sin\theta}{\cos^2\theta}\equiv\frac{2\sin^2\theta}{\sin\theta\cos^2\theta}$ $\equiv\frac{2}{\sin\theta}\times\frac{\sin^2\theta}{\cos^2\theta}\equiv\frac{2\operatorname{cosec}\theta}{\cot^2\theta}=\text{LHS}$ or e.g. QED or e.g. Proven	A1*	2.1
		(3)	

Part (a) Notes

(a) **Condone a complete proof <u>entirely</u> in *x* (or another variable) instead of θ**
Condone "=" for "≡"

Note that we are marking this as **B1M1A1** not **M1M1A1**

B1: Adds the fractions to obtain a **correct** single fraction (not fractions over fractions) in any form. Condone missing brackets when they combine their fractions as long as they are recovered to give a correct fraction.

This can be done in a variety of ways but when combined, the fraction must be **correct** e.g.

$$\frac{\operatorname{cosec}\theta+1+\operatorname{cosec}\theta-1}{(\operatorname{cosec}\theta-1)(\operatorname{cosec}\theta+1)} \text{ or } \frac{2\operatorname{cosec}\theta}{(\operatorname{cosec}\theta-1)(\operatorname{cosec}\theta+1)} \text{ or } \frac{2\operatorname{cosec}\theta}{\operatorname{cosec}^2\theta-1}$$

$$\text{or e.g. } \left(\frac{1}{\frac{1}{\sin\theta}-1}+\frac{1}{\frac{1}{\sin\theta}+1}=\frac{\sin\theta}{1-\sin\theta}+\frac{\sin\theta}{1+\sin\theta}\right)=\frac{2\sin\theta}{1-\sin^2\theta} \text{ etc.}$$

M1: **Uses** a **correct** Pythagorean identity anywhere in their attempt e.g. $\operatorname{cosec}^2\theta-1=\cot^2\theta$, $\sin^2\theta+\cos^2\theta=1$ etc. or equivalent

A1*: Correct work with all necessary steps shown leading to the given answer. See scheme for the necessary steps. They need to proceed via sine and cosine to the given answer. There should be **no notational or bracketing errors and no mixed or missing variables**. E.g. we would consider $\cos^2\theta$ written as $\cos\theta^2$ a notational error.
Condone reaching $2\sec\theta\tan\theta$*

Way 2 (Meet in the middle)

B1: See Way 1

M1: See Way 1

A1*: Correct work on the RHS with all necessary steps shown leading to showing the equivalence with the LHS. See scheme for the necessary steps. There should be no notational or bracketing errors and no mixed or missing variables.
For this approach there must be a (minimal) conclusion e.g. "= LHS", "QED", "Hence proven" etc.

It is possible to start with the rhs e.g.:

$$2\tan\theta\sec\theta = 2\frac{\sin\theta}{\cos\theta}\times\frac{1}{\cos\theta}$$

$$= \frac{2\sin\theta}{\cos^2\theta} = \frac{2\operatorname{cosec}\theta}{\cot^2\theta}$$

$$= \frac{2\operatorname{cosec}\theta}{\operatorname{cosec}^2\theta - 1} = \frac{2\operatorname{cosec}\theta}{(\operatorname{cosec}\theta - 1)(\operatorname{cosec}\theta + 1)}$$

$$= \frac{\operatorname{cosec}\theta + 1 + \operatorname{cosec}\theta - 1}{(\operatorname{cosec}\theta - 1)(\operatorname{cosec}\theta + 1)}$$

$$= \frac{1}{\operatorname{cosec}\theta - 1} + \frac{1}{\operatorname{cosec}\theta + 1}$$

B1: Correctly reaches $2\tan\theta\sec\theta = \dfrac{2\operatorname{cosec}\theta}{(\operatorname{cosec}\theta - 1)(\operatorname{cosec}\theta + 1)}$

M1: **Uses** a **correct** Pythagorean identity anywhere in their attempt e.g. $\operatorname{cosec}^2\theta - 1 = \cot^2\theta$, $\sin^2\theta + \cos^2\theta = 1$ etc. or equivalent

A1*: Correct work with all necessary steps shown leading to the lhs. See scheme for the necessary steps. There should be no notational or bracketing errors and no mixed or missing variables.

8(b)	$2\tan 2x\sec 2x=\cot 2x\sec 2x$	B1	2.2a
	$2\tan 2x\sec 2x-\cot 2x\sec 2x=0$ $\Rightarrow \sec 2x\left(2\tan 2x-\cot 2x\right)=0$ $2\tan 2x-\cot 2x=0\Rightarrow 2\tan 2x=\cot 2x\Rightarrow \tan^2 2x=\dfrac{1}{2}$ **or** $2\tan 2x-\cot 2x=0\Rightarrow 2\dfrac{\sin 2x}{\cos 2x}=\dfrac{\cos 2x}{\sin 2x}\Rightarrow 2\sin^2 2x=\cos^2 2x$ $\Rightarrow 2\sin^2 2x=1-\sin^2 2x\Rightarrow \sin^2 2x=\dfrac{1}{3}$ **or** $2\left(1-\cos^2 2x\right)=\cos^2 2x\Rightarrow \cos^2 2x=\dfrac{2}{3}$ **or** $2\tan 2x=\cot 2x\Rightarrow \dfrac{4\tan x}{1-\tan^2 x}=\dfrac{1-\tan^2 x}{2\tan x}$ $\Rightarrow \tan^4 x-10\tan^2 x+1=0$ **or** $2\tan 2x-\cot 2x=0\Rightarrow 2\dfrac{\sin 2x}{\cos 2x}=\dfrac{\cos 2x}{\sin 2x}\Rightarrow 2\sin^2 2x=\cos^2 2x$ $2\sin^2 2x=\cos^2 2x\Rightarrow 1-\cos 4x=\dfrac{1}{2}\left(\cos 4x+1\right)\Rightarrow \cos 4x=\dfrac{1}{3}$	M1	2.1
	$2x=\tan^{-1}\dfrac{1}{\sqrt{2}}=K\Rightarrow x=\dfrac{K}{2}$ **or** $2x=\sin^{-1}\dfrac{1}{\sqrt{3}}=K\Rightarrow x=\dfrac{K}{2}$ **or** $2x=\cos^{-1}\sqrt{\dfrac{2}{3}}=K\Rightarrow x=\dfrac{K}{2}$ **or** $\tan^2 x=5\pm 2\sqrt{6}\Rightarrow x=\tan^{-1}\left(\sqrt{5\pm 2\sqrt{6}}\right)$ **or** $\cos 4x=\dfrac{1}{3}\Rightarrow 4x=\cos^{-1}\left(\dfrac{1}{3}\right)=K\Rightarrow x=\dfrac{1}{4}K$	M1	1.1b
	$x=17.6°,\ 72.4°$	A1	1.1b
		(4)	
			(7 marks)
	(b) Notes		

(b) Note that attempts solve an equation of the form:

$2\tan x\sec x=\cot 2x\sec 2x$ or e.g. $2\tan\theta\sec\theta=\cot 2\theta\sec 2\theta$ or e.g. $2\tan\theta\sec\theta=\cot 2x\sec 2x$

Will generally score no marks in part (b)

Condone the use of θ instead of x here.

B1: Deduces the correct equation using the result from part (a)

M1: Factors out or cancels the $\sec 2x$ to obtain $...\tan 2x\pm...\cot 2x=0$ oe e.g. $...\tan 2x=\pm...\cot 2x$ leading to an equation of the form:

$\tan^2 2x=\alpha$ oe e.g. $\cot^2 2x=\dfrac{1}{\alpha}$ where $\alpha>0$

Or uses $\tan 2x=\dfrac{\sin 2x}{\cos 2x}$ and $\cot 2x=\dfrac{\cos 2x}{\sin 2x}$ and then $\sin^2 2x=\pm1\pm\cos^2 2x$ or $\cos^2 2x=\pm1\pm\sin^2 2x$ to obtain an equation of the form $\sin^2 2x=\beta$ or $\cos^2 2x=\gamma$ oe e.g. $\text{cosec}^2 2x=\dfrac{1}{\beta}$ or $\sec^2 2x=\dfrac{1}{\gamma}$ where $0<\beta<1$ or $0<\gamma<1$

Or uses $\tan 2x=\dfrac{2\tan x}{1-\tan^2 x}$ and $\cot 2x=\dfrac{1-\tan^2 x}{2\tan x}$ to obtain a 3TQ in $\tan^2 x$ (or possibly in $\sec^2 x$)

Or uses $\tan 2x=\dfrac{\sin 2x}{\cos 2x}$ and $\cot 2x=\dfrac{\cos 2x}{\sin 2x}$ and then $2\sin^2 2x=\pm1\pm\cos 4x$ and $2\cos^2 2x=\pm1\pm\cos 4x$ to obtain an equation of the form $\cos 4x=k,\quad 0<k<1$

M1: Correct order of operations from $\tan^2 2x=\alpha$ or $\sin^2 2x=\beta$ or $\cos^2 2x=\gamma$ or $\cos 4x=k$ or equivalents e.g. $\text{cosec}^2 2x=\dfrac{1}{\beta}$ where $\alpha>0$ or $0<\beta<1$ or $0<\gamma<1$ or $0<k<1$ leading to at least one value for x e.g. square roots, finds inverse tan/sin/cos/cosec and divides by 2 or inverse cos and divides by 4

or from $\tan^2 x=k,\ k>0$ which follows their equation (may need to check) and then finds $x=\tan^{-1}\sqrt{k}$

You may need to check their value(s) (in degrees or radians) to see if the correct order of operations has been used. May be implied by e.g. $17.6°$ or $17.7°$ provided no incorrect work is seen.

A1: Correct values. Allow awrt $17.6°$ and awrt $72.4°$. The degrees symbol is not required. Ignore any values outside the range (correct or incorrect) but if there are extra angles in range score A0. Answers in radians score A0.

Note that some candidates may convert to $\sin x$ or $\cos x$ and then solve:

E.g.

$$\tan^2 2x=\frac{1}{2}\Rightarrow\frac{\sin^2 2x}{\cos^2 2x}=\frac{1}{2}\Rightarrow\frac{4\sin^2 x\cos^2 x}{\left(2\cos^2 x-1\right)^2}\rightarrow 12\cos^4 x-12\cos^2 x+1=0$$

$$\text{or }\frac{4\sin^2 x\cos^2 x}{\left(1-2\sin^2 x\right)^2}\rightarrow 12\sin^4 x-12\sin^2 x+1=0$$

$$\Rightarrow\cos^2 x/\sin^2 x=\frac{3\pm\sqrt{6}}{6}\Rightarrow\cos x/\sin x=\pm\sqrt{\frac{3\pm\sqrt{6}}{6}}\Rightarrow x=17.6°,\ 72.4°$$

These can be marked in a similar way.

Alternative not using part (a):

$$\frac{1}{\text{cosec}2x-1}+\frac{1}{\text{cosec}2x+1}=\cot 2x\sec 2x$$

$$\Rightarrow \frac{2\text{cosec}2x}{\text{cosec}^2 2x-1}=\cot 2x\sec 2x$$

$$\Rightarrow \frac{2\text{cosec}2x}{\cot^2 2x}=\frac{\cos 2x}{\sin 2x}\times\frac{1}{\cos 2x}=\text{cosec}2x$$

$$\Rightarrow 2\tan^2 2x=1\Rightarrow \tan^2 2x=\frac{1}{2}$$

Score as:

M1: For correct work leading to one of the forms in the main scheme e.g.

$$\tan^2 2x=\alpha \text{ oe e.g. } \cot^2 2x=\frac{1}{\alpha} \text{ where } \alpha>0$$

or

$$\sin^2 2x=\beta \text{ or } \cos^2 2x=\gamma \text{ oe e.g. } \text{cosec}^2 2x=\frac{1}{\beta} \text{ or } \sec^2 2x=\frac{1}{\gamma}$$

where $0<\beta<1$ or $0<\gamma<1$

B1: Any correct equation e.g. $\tan^2 2x=\frac{1}{2}$, $\cot^2 2x=2$, $\sin^2 2x=\frac{1}{3}$ etc.

Then **M1A1** as main scheme