Question	Scheme	Marks	AOs
9(a) Way 1	$H = \pm ax^2 \pm bx \pm c$ $x=0, H=2 \Rightarrow c=2$ **and** either $x=20, H=0.8 \Rightarrow 0.8 = 400a + 20b + 2$ **or** $H = ax^2 + bx + c \Rightarrow \frac{\mathrm{d}H}{\mathrm{d}x} = 2ax + b$ $x=9, \frac{\mathrm{d}H}{\mathrm{d}x} = 0 \Rightarrow 18a + b = 0$	M1	3.3
	$H = \pm ax^2 \pm bx \pm c$ $x=0, H=2 \Rightarrow c=2$ **and** $x=20, H=0.8 \Rightarrow 0.8 = 20^2 a + 20b + 2$ **and** $H = ax^2 + bx + c \Rightarrow \frac{\mathrm{d}H}{\mathrm{d}x} = 2ax + b$ $x=9, \frac{\mathrm{d}H}{\mathrm{d}x} = 0 \Rightarrow 18a + b = 0$	dM1	3.1b
	$0.8 = 400a + 20b + 2,\ 18a + b = 0 \Rightarrow a = ..., b = ...$	ddM1	1.1b
	$H = -0.03x^2 + 0.54x + 2$	A1	2.2a
		(4)	

(a) Way 1 Notes

Condone use of *y* for *H* for the <u>method</u> marks.

A model of the form $H = x^2 + ax + b$ or $H = -x^2 + ax + b$ **will score no marks.**

Note that it is possible to identify (by symmetry) that the points (–2, 0.8) and (18, 2) also lie on the parabola so you may see valid use of these points.

M1: Uses the equation $H = \pm ax^2 \pm bx \pm c$ to model the path and uses $x = 0$ and $H = 2$ **correctly placed** to establish the value of the constant term and uses $x = 20$ and $H = 0.8$ **or** $x = 9, \frac{\mathrm{d}H}{\mathrm{d}x} = 0$ to give an equation in '*a*' and '*b*' with $\frac{\mathrm{d}H}{\mathrm{d}x}$ of the form $...\alpha x + \beta$

An alternative is to recognise that the maximum occurs when $x = -\frac{b}{2a} = 9$ or equivalent

e.g. maximum when $x = 9 \Rightarrow H = a(x-9)^2 + ... = ax^2 - 18ax + ... \Rightarrow b = -18a$

Award for $\pm\frac{b}{2a} = 9$ or equivalent.

They may also use e.g. (–2, 0.8) or (18, 2) to give an equation in *a* and *b*.

dM1: This mark requires:

- uses the equation $H = \pm ax^2 \pm bx \pm c$ to model the path and uses $x = 0$ and $H = 2$ **correctly placed** to establish the value of the constant term
- uses $x = 20$ and $H = 0.8$ **correctly placed and** $x = 9, \frac{\mathrm{d}H}{\mathrm{d}x} = 0$ to give 2 equations in '*a*' and '*b*' with $\frac{\mathrm{d}H}{\mathrm{d}x}$ of the form $...ax + b$ or as above using $\pm\frac{b}{2a} = 9$

They may also use e.g. (–2, 0.8) or (18, 2) to give an equation in *a* and *b*.

ddM1: Solves their 2 equations in "*a*" and "*b*" to find their '*a*' and '*b*'.
This may be done on a calculator. You do **not** need to check their method for solving.

A1: Correct **equation**. Must be $H = \mathrm{f}(x)$.

(a)	$x = 9 \text{ at max} \Rightarrow H = A \pm B(x-9)^2$		
Way 2	**and** either $x = 0, H = 2 \Rightarrow 2 = A + 81B$ **or** $x = 20, H = 0.8 \Rightarrow 0.8 = A + 121B$	M1	3.3
	$x = 9 \text{ at max} \Rightarrow H = A + B(x-9)^2$ **and** $x = 0, H = 2 \Rightarrow 2 = A + 81B$ **and** $x = 20, H = 0.8 \Rightarrow 0.8 = A + 121B$	dM1	3.1b
	$2 = A + 81B,\ 0.8 = A + 121B \Rightarrow A = 4.43,\ B = -0.03$	ddM1	1.1b
	$H = 4.43 - 0.03(x-9)^2$	A1	2.2a
		(4)	

(a) Way 2 Notes

Condone use of *y* for *H* for the <u>method</u> marks.
A model of the form $H = A \pm (x-9)^2$ **will score no marks.**

M1: Uses the equation $H = A \pm B(x-9)^2$ or $H = A \pm B(9-x)^2$ to model the path and uses one of the 'end points' **correctly placed** to give an equation in 'A' and 'B'
They may also use e.g. (–2, 0.8) or (18, 2) to give an equation in A and B.

dM1: Uses the equation $H = A + B(x-9)^2$ or $H = A + B(9-x)^2$ to model the path and uses both 'end points' **correctly placed** to give 2 equations in 'A' and 'B'
They may also use e.g. (–2, 0.8) or (18, 2) to give an equation in A and B.

ddM1: Solves their 2 equations in "A" and "B" to find their 'A' and 'B'.
This may be done on a calculator. You do **not** need to check their method for solving.

A1: Correct **equation**. Must be $H = \mathrm{f}(x)$.

Note that using $H = A + B(x-9)^2$ followed by the incorrect assumption that $A = 2$ is unlikely to score any marks as they will subsequently not be able to produce 2 equations in "A" and "B"

<u>**Possible alternative 3:**</u>

$$H = A\left((x-9)^2 - 81\right) + B$$
$$x = 0, H = 2 \Rightarrow B = 2$$
$$x = 20, H = 0.8 \Rightarrow 0.8 = 40A + B$$
$$B = 2 \Rightarrow A = -0.03$$
$$H = 2 - 0.03\left((x-9)^2 - 81\right)$$

M1: Uses the equation $H = A\left((x-9)^2 - 81\right) + B$ to model the path and uses $H = 2$ when $x = 0$ **correctly placed** to find "B"

dM1: Uses the equation $H = A\left((x-9)^2 - 81\right) + B$ to model the path and uses $H = 0.8$ when $x = 20$ **correctly placed.** May also use e.g. (–2, 0.8) or (18, 2)

ddM1: Substitutes their value for "B" to find a value for "A"

A1: Correct **equation**. Must be $H = \mathrm{f}(x)$.

(b)	Examples must focus on why the model may not be appropriate or give situations where the model would break down e.g.: • H is unlikely to be a quadratic function in x • The path is unlikely to be parabolic • Wind may affect the path of the ball • Wind may affect the distance the ball travels • Air resistance has not been considered • The ball is unlikely to travel in a vertical plane (as it may spin) • The ball is not a particle so has dimensions/size • The ground is unlikely to be horizontal • There may be trees (or other hazards) that would affect the path of the ball • The shape of the ball may affect the motion Condone statements (where the link to the model is not completely made) such as • The ball will spin • Ground is not flat • The ball is not a particle Do **not** accept statements that refer to the situation outside the range of the throw e.g. • The model is not valid for all values of x • H will become negative Do **not** accept statements that do not refer to the given model or single word vague answers e.g. • The distances may have been measured incorrectly • The ball is not modelled as a particle • "Friction", "Spin", "Force", "air resistance" • It does not take into account the weight of the ball • It depends how good the thrower is • You cannot throw the ball the same way every time	B1	3.5b
		(1)	
(c)	$x=16 \Rightarrow H=-0.03(16)^2+0.54(16)+2=\ldots$	M1	3.4
	$H = 2.96$ So Chandra would not be able to catch the ball	A1	3.2a
		(2)	
			(7 marks)

Notes for (b) and (c)

(b)

B1: Gives a suitable limitation – see scheme
If more than one limitation is given and one is acceptable then award this mark as long as none of the other statements are contradictory (they may be incorrect/inappropriate)

(c)

M1: Substitutes $x = 16$ into their equation modelling the path **to obtain a value** for H. This may be seen explicitly as above or may be implied by their value (you may need to check). **Must have a quadratic function in *x*.**

A1: Depends on

- A correct equation
- $H = 2.96$
- Correct conclusion that she cannot catch the ball or equivalent

A minimum for M1A1 could be e.g. $x = 16 \Rightarrow H = 2.96$ "so no"

(c) Alternative:

$$\text{e.g. } 2.5 = 4.43 - 0.03(x-9)^2 \Rightarrow x = 9 + \frac{\sqrt{579}}{3} = 17.02...$$

So Chandra would not be able to catch the ball

M1: Substitutes $H = 2.5$ into their **quadratic** equation modelling the path **to obtain a value** for x. This may be seen explicitly as above or may be implied by their value (you may need to check). **Must have a quadratic function in *x*.**

A1: Depends on

- A correct equation
- $x =$ awrt 17
- Correct conclusion that she cannot catch the ball or equivalent.

A minimum for M1A1 could be e.g. $H = 2.5 \Rightarrow x = 17$ "so no"