

Question	Scheme	Marks	AOs
11	$\int 8x^2 e^{-3x} dx = -\frac{8x^2}{3} e^{-3x} + \int \frac{16x}{3} e^{-3x} dx$	M1 A1	2.1 1.1b
	$= -\frac{8x^2}{3} e^{-3x} - \frac{16x}{9} e^{-3x} + \int \frac{16}{9} e^{-3x} dx$	dM1	1.1b
	$\left[ -\frac{8x^2}{3} e^{-3x} - \frac{16x}{9} e^{-3x} - \frac{16}{27} e^{-3x} \right]_0^1$ $= -\frac{8}{3} e^{-3} - \frac{16}{9} e^{-3} - \frac{16}{27} e^{-3} - \left( -0 - 0 - \frac{16}{27} \right)$	M1	2.1
	$= \frac{16}{27} - \frac{136}{27} e^{-3}$	A1	1.1b
		(5)	

(5 marks)

### Notes

Mark positively in this question and do not penalise poor notation such as a missing “dx” or spurious integral signs, “+ c” etc. as long as the intention is clear.

**M1:** Obtains  $\pm \alpha x^2 e^{-3x} \pm \beta \int x e^{-3x} dx$

(you do not need to be concerned about how they arrive at this)

**A1:** Correct expression **simplified or unsimplified**. E.g. allow  $-\frac{8x^2}{3} e^{-3x} - \int -\frac{16x}{3} e^{-3x} dx$

Note that we condone the “8” missing for this mark so allow e.g.  $-\frac{x^2}{3} e^{-3x} - \int -\frac{2x}{3} e^{-3x} dx$

Note that notation may be poor here but the intention clear e.g. if they obtain

$-\frac{8x^2}{3} e^{-3x} + \left[ \frac{16x}{3} e^{-3x} \right]$  and then attempt to integrate  $\frac{16x}{3} e^{-3x}$  both marks can be implied.

**dM1:** Attempts parts again on  $\pm \beta \int x e^{-3x} dx$  to obtain  $\pm A x e^{-3x} \pm B \int e^{-3x} dx$

This may be seen in isolation and does not need to be seen as part of the complete integration. **Depends on the first method mark.**

Watch for the DI method (with or without the 8):

	D	I
+	$8x^2$	$e^{-3x}$
-	$16x$	$-\frac{1}{3} e^{-3x}$
+	$16$	$\frac{1}{9} e^{-3x}$
-	$0$	$-\frac{1}{27} e^{-3x}$

Giving the correct integration e.g.  $\int 8x^2 e^{-3x} dx = -\frac{8x^2}{3} e^{-3x} - \frac{16x}{9} e^{-3x} - \frac{16}{27} e^{-3x}$

In such cases score M1dM1 for obtaining  $\pm px^2 e^{-3x} \pm qx e^{-3x} \pm re^{-3x}$ ,  $p, q, r \neq 0$  and then A1 for the **correct** first 2 terms, with or without the factor of 8.

Note that for this approach M1A1dM0 is not possible.

**M1:** Substitutes the limits 1 and 0 into an expression of the form  $\pm \alpha x^2 e^{-3x} \pm \beta x e^{-3x} \pm \gamma e^{-3x}$ ,  $\alpha, \beta, \gamma \neq 0$  and subtracts the right way round.

Must see evidence of the use of **both** limits and subtraction and use of  $e^0 = 1$ .

Note that some candidates apply the limits as they go e.g. to the  $\left[ -\frac{8x^2}{3} e^{-3x} \right]$  which is

acceptable but you will need to check carefully that overall they are satisfying the conditions above.

Condone not realising that the first 2 terms evaluate to 0 when substituting  $x = 0$  e.g.

condone  $-\frac{8}{3}e^{-3} - \frac{16}{9}e^{-3} - \frac{16}{27}e^{-3} - \left( -\frac{8}{3} - \frac{16}{9} - \frac{16}{27} \right)$  as we have evidence of  $e^0 = 1$

Note that e.g.  $-\frac{8}{3}e^{-3} - \frac{16}{9}e^{-3} - \frac{16}{27}e^{-3} - \left( -\frac{16}{27}e^0 \right) = -\frac{136}{27}e^{-3} - \frac{16}{27}$  scores M0 as it suggests that  $e^0 = -1$  not +1.

**A1:** Correct answer of  $\frac{16}{27} - \frac{136}{27}e^{-3}$  but allow equivalent exact fractions and condone

$\frac{16}{27} - \frac{136}{27e^3}$ . Isw once the correct answer is seen.

**Candidates who consistently misread  $8x^2e^{-3x}$  as  $8x^2e^{3x}$ :**

$$\begin{aligned} \int 8x^2 e^{3x} dx &= \frac{8x^2}{3} e^{3x} - \int \frac{16x}{3} e^{3x} dx \\ &= \frac{8x^2}{3} e^{3x} - \frac{16x}{9} e^{3x} + \int \frac{16}{9} e^{3x} dx \\ &= \left[ \frac{8x^2}{3} e^{3x} - \frac{16x}{9} e^{3x} + \frac{16}{27} e^{3x} \right]_0^1 \\ &= \frac{8}{3}e^3 - \frac{16}{9}e^3 + \frac{16}{27}e^3 - \left( \frac{16}{27} \right) = \frac{40}{27}e^3 - \frac{16}{27} \end{aligned}$$

Scores a maximum of M1A0dM1M1A0

The main scheme can be applied similarly e.g.

**M1:** Attempts parts to obtain  $\alpha x^2 e^{3x} - \beta \int x e^{3x} dx$ ,  $\alpha, \beta > 0$

**A0:** Not available

**dM1:** Attempts parts again on  $\beta \int x e^{3x} dx$  to obtain  $Cx e^{3x} - D \int e^{3x} dx$ ,  $C, D > 0$

**M1:** Substitutes the limits 1 and 0 into an expression of the form  $\pm \lambda x^2 e^{3x} \pm \mu x e^{3x} \pm \gamma e^{3x}$ ,  $\lambda, \mu, \gamma \neq 0$  and subtracts the right way round.

Must see evidence of the use of **both** limits and subtraction and use of  $e^0 = 1$ .

**A0:** Not available

But note, do **not** allow mixing of  $3x$ 's and  $-3x$ 's. If there are a mixture, apply the main scheme.