Question	Scheme	Marks	AOs
12(a)	$\frac{1}{V(25-V)} = \frac{P}{V} + \frac{Q}{25-V}$ e.g. $1 = P(25-V) + QV$ $V = 0$ or $V = 25$ leading to $P = \ldots$ or $Q = \ldots$	M1	1.1b
	$\frac{1}{V(25-V)} = \frac{1}{25V} + \frac{1}{25(25-V)}$	A1	1.1b
		(2)	

Notes

(a)

M1: Sets $\frac{1}{V(25-V)} = \frac{P}{V} + \frac{Q}{25-V}$ and uses a correct method to identify the value of at least one constant.

Do **not** condone incorrect work e.g. $\frac{1}{V(25-V)} = \frac{P}{V} + \frac{Q}{25-V} \Rightarrow 1 = PV + Q(25-V)$ etc. this scores M0

A1: Correct partial fractions in any form e.g.

$\frac{1}{25V} + \frac{1}{25(25-V)}, \frac{1}{25V} + \frac{1}{625-25V}, \frac{\frac{1}{25}}{V} + \frac{\frac{1}{25}}{(25-V)}, \frac{1}{25V} - \frac{1}{25(V-25)}, \frac{1}{25}\left(\frac{1}{V} + \frac{1}{25-V}\right)$ etc.

Note that this mark is not just for the correct constants, it is for the correctly written fractions either seen in part (a) **or used in part (b).** Allow 0.04 for $\frac{1}{25}$.

Correct partial fractions only scores both marks.

If the correct fractions are obtained following incorrect work score M0A0 but allow full recovery in the rest of the question.

(b) Way 1	$\int \frac{1}{V(25-V)}\,\mathrm{d}V = \int \frac{1}{25V}+\frac{1}{25(25-V)}\,\mathrm{d}V = \frac{1}{25}\ln V - \frac{1}{25}\ln(25-V)$	M1	3.1a
	$\frac{1}{25}\ln V - \frac{1}{25}\ln(25-V) = \frac{1}{10}t\,(+c)$	A1ft	1.1b
	$t=0, V=20 \Rightarrow \frac{1}{25}\ln 20 - \frac{1}{25}\ln(25-20) = c \left(\Rightarrow c = \frac{1}{25}\ln 4\right)$	M1	3.4
	$V=24 \Rightarrow t = \frac{2}{5}\ln 24 - \frac{2}{5}\ln(25-24) - \frac{2}{5}\ln 4$	**d**M1	3.1b
	= 43 (or exact $24\ln 6$)	A1	3.2a
		(5)	
	Alternative for the final 3 marks:		
	$\left[\frac{1}{25}\ln V - \frac{1}{25}\ln(25-V)\right]_{20}^{24} = \left[\frac{1}{10}t\right]_0^T \Rightarrow \frac{1}{25}\ln 24 - \frac{1}{25}\ln 4 = \frac{1}{10}T$	M1	3.4
	$T = \frac{10}{25}\ln 24 - \frac{10}{25}\ln 4 = \ldots$	**d**M1	3.1b
	= 43 (or exact $24\ln 6$)	A1	3.2a
(c)	$\frac{1}{25}\ln V - \frac{1}{25}\ln(25-V) = \frac{1}{10}t + \frac{1}{25}\ln 4$ $\ln V - \ln(25-V) = 2.5t + \ln 4$ $\ln\frac{V}{4(25-V)} = 2.5t \Rightarrow \frac{V}{4(25-V)} = \mathrm{e}^{2.5t}$	M1	2.1
	$\Rightarrow V = 4\mathrm{e}^{2.5t}(25-V) \Rightarrow V + 4V\mathrm{e}^{2.5t} = 100\mathrm{e}^{2.5t} \Rightarrow V = \ldots$	M1	2.1
	$\Rightarrow V = \frac{100\mathrm{e}^{2.5t}}{1+4\mathrm{e}^{2.5t}} = \frac{100}{\mathrm{e}^{-2.5t}+4}$	A1	1.1b
		(3)	
(d)	25 (microlitres)	B1	2.2a
	Since e.g. As $t \to \infty$, $\mathrm{e}^{"-2.5"t} \to 0$	B1	2.4
		(2)	
			(12 marks)

Notes

(b) **Mark (b) and (c) together**

M1: Realises that $\int \frac{\ldots}{V(25-V)}\,(\mathrm{d}V)$ is required and reaches the form $p\ln\alpha V \pm q\ln\beta(25-V)$ (or e.g. $p\ln\alpha V \pm q\ln\beta(V-25)$) or equivalent for this integration e.g. $p\ln 25V - q\ln(625-25V)$ with p and q non-zero.

But note that $\int \frac{\ldots}{V(25-V)}\mathrm{d}V = \ln V(25-V)$ does not score this mark unless we see an attempt to integrate the partial fractions first.

Condone missing brackets e.g. around the $V-25$ for this mark

Note that the rhs may be incorrect or missing for this mark.

A1ft: Fully correct **equation** following through their P and Q. The "+ c" is **not** required here. You may need to check carefully when awarding this mark as there will be various alternative correct (or correct ft) forms e.g. these are correct (for correct PF's):

$\frac{1}{25}\ln 25V - \frac{1}{25}\ln(625-25V) = \frac{1}{10}t(+c)$, $\ln V - \ln(25-V) = 2.5t(+c)$,

$\frac{2}{5}\ln 25V - \frac{2}{5}\ln(25-V) = t(+c)$, $\frac{2}{5}\ln 5V - \frac{2}{5}\ln(125-5V) = t(+c)$

In general look for an equation of the form $P\ln\alpha V - Q\ln\beta(25-V) = \frac{1}{10}t(+c)$ or a multiple of this equation. Do **not** condone missing brackets unless they are implied by later work e.g. $\ln 25-V$ for $\ln(25-V)$

Allow $(\ )$ or $|\ |$ around the arguments of the ln's and condone "log" for ln.

M1: States or uses $t = 0$ and $V = 20$ consistently leading to a constant of integration which may be simplified or unsimplified. May be implied by their constant so may need to be checked.
This mark is not formally dependent but depends on having made some attempt to integrate both sides, however poor.

dM1: States or uses $V = 24$ and proceeds to find a value for t (even if $t < 0$). You do not need to check the processing provided they reach a value for t. **Depends on the previous method mark** and depends on an attempt to integrate both sides however poor.
May be implied by their value for t so may need to be checked.

A1: Correct value of 43 or awrt 43.0 or exact value of 24ln6.
Units are not required but if any are given it must be minutes or condone "m".
Note that in hours the time is 0.7167037877… and scores A0

Alternative for final 3 marks:

M1: $\left[\frac{1}{25}\ln V - \frac{1}{25}\ln(25-V)\right]_{20}^{24} = \left[\frac{1}{10}t\right]_0^T \Rightarrow \frac{1}{25}\ln 24 - \frac{1}{25}\ln 4 = \frac{1}{10}T$

Applies the limits 20 and 24 to lhs and 0 to "T" or e.g. "t" on rhs
This mark is not formally dependent but depends on having made some attempt to integrate both sides, however poor.

dM1: $T = \frac{10}{25}\ln 24 - \frac{10}{25}\ln 4 = ...$ Solves to find "T". You do not need to check the processing provided they reach a value for t/T. **Depends on the previous method mark** and depends on having made some attempt to integrate both sides however poor.

A1: Correct value of 43 or awrt 43.0 or exact value of 24ln6.
Units are not required but if any are given it must be minutes or condone "m".
Note that in hours the time is 0.7167037877… and scores A0

Note

$$\int \frac{1}{25V} + \frac{1}{25(25-V)}\,dV = \int \frac{1}{25V} - \frac{1}{25V-625}\,dV = \frac{1}{25}\ln V - \frac{1}{25}\ln(25V-625) = \frac{1}{10}t(+c)$$

is also correct integration and scores M1A1

The subsequent marks are also available as described above and could lead to the correct answer if the limits and constant of integration are dealt with correctly.
Use review for any examples like these if you are unsure but generally apply the MS as above.

(c) The marks in (c) depend on having integrated their partial fractions to obtain an equation of the form $\pm...\ln...V \pm ...\ln...(25-V) = \pm kt \pm c$, $k, c \neq 0$ and … are non-zero constants or equivalent if they have already attempted to eliminate the ln's in (b)

e.g. $\frac{...V}{...(25-V)} = e^{\pm kt \pm c}$ oe

M1: Uses fully correct log work, having obtained a constant of integration, to eliminate all the ln's including from e.g. $e^{\ln 4}$. We condone sign or coefficient slips only.

M1: Proceeds from an **equation of the form** $\frac{...V}{...(25-V)} = ...e^{...t}$ oe using correct algebra to

$V = \ldots$ e.g. $\frac{...V}{...(25-V)} = ...e^{...t} \Rightarrow ...V = ...(25-V)...e^{...t} \Rightarrow (...\pm...)V = ...e^{...t} \Rightarrow V = ...$

Condone sign/coefficient slips only.

A1: Correct expression not just values for the constants.

(d)

B1: Correct value of 25 seen
Allow e.g. < 25 or „ 25
Condone " > 25" but the following mark is then not available

B1: Depends on a correct final equation in any form in (c) e.g. $V = \frac{100e^{2.5t}}{1+4e^{2.5t}}$ oe **and one of**:

- Considers the behaviour as $t \to \infty$ e.g. states that as $t \to \infty$, $e^{-2.5t} \to 0$ (condone "= 0") oe
- $V < 25$ as $\ln(25 - V)$ is not possible when $V \ldots 25$
- Verifies the 25 using a value of t, $t \ldots 9$

Using the differential equation:

B1: Correct value of 25 seen
Allow e.g. " < 25" or „ 25
Condone " > 25" but the following mark is then not available

B1: E.g. when $V = 25$, $\frac{dV}{dt} = 0$ **or** $\frac{dV}{dt} < 0$ if $V > 25$