Question	Scheme	Marks	AOs
14(a)(i)	Centre is (3, −7)	B1	1.1b
(ii)	$(x-3)^2+(y+7)^2=49+9-33 \Rightarrow r^2=...(25)$	M1	1.1b
	$r=5$	A1	1.1b
		(3)	
(b)	Distance between centres = $\sqrt{(3+6)^2+(-7+8)^2}=\sqrt{82}$	M1 A1ft	3.1a 1.1b
	$"\sqrt{82}"-"5"$ **or** $"\sqrt{82}"+"5"$	dM1	3.1a
	$\sqrt{82}-5$ **and** $\sqrt{82}+5$	A1	2.2a
	$\{k:\sqrt{82}-5<k\}\cap\{k:k<\sqrt{82}+5\}$ or e.g. $\{k:\sqrt{82}-5<k<\sqrt{82}+5\}$	A1	2.5
		(5)	
		(8 marks)	

Notes

(a)(i)

B1: Correct centre. Allow as a coordinate pair or written separately e.g. $x = 3$, $y = -7$ or as a column vector $\begin{pmatrix} 3 \\ -7 \end{pmatrix}$

Condone missing brackets e.g. 3, −7 but do **not** allow coordinates the wrong way round.

(ii)

M1: Uses a correct strategy to find the radius or radius^2

Requires an attempt at: $(x\pm3)^2+(y\pm7)^2-3^2-7^2\pm33=0 \Rightarrow (x\pm3)^2+(y\pm7)^2=\alpha,\ \alpha>0$

Award for $(x\pm3)^2+(y\pm7)^2-a^2-b^2\pm33=0 \Rightarrow (x\pm3)^2+(y\pm7)^2=\alpha,\ \alpha>0$ with at least one of $a = 3$ or $b = 7$ (or 9 or 49)

You may see an attempt at $"f^2+g^2-c"$ or $\sqrt{"f^2+g^2-c"}$ e.g. $"3^2+7^2\pm33"$ or $\sqrt{"3^2+7^2\pm33"}$

A1: Correct radius of 5. Do **not** allow ±5 or $\sqrt{25}$.

May be scored following $(x\pm3)^2+(y\pm7)^2=25$

Correct answers only in (a) scores B1M1A1

(b)

M1: Uses Pythagoras correctly on their centre from part (a) and the given centre to find the distance between the centres.

Look for $\sqrt{(-6-(\text{their } x))^2+(-8-(\text{their } y))^2}$ or e.g. $\sqrt{((\text{their } x)-(-6))^2+((\text{their } y)-(-8))^2}$ but condone one sign slip with their coordinates if the intention is clear.

A1ft: Correct distance or follow through their centre from part (a).
This may be implied by their value. Condone the use of decimals so allow 3sf accuracy e.g. awrt 9.06 for $\sqrt{82}$ or you may need to check their value following an incorrect centre in (a)(i). Not e.g. $\pm\sqrt{82}$ unless the positive root is subsequently used.

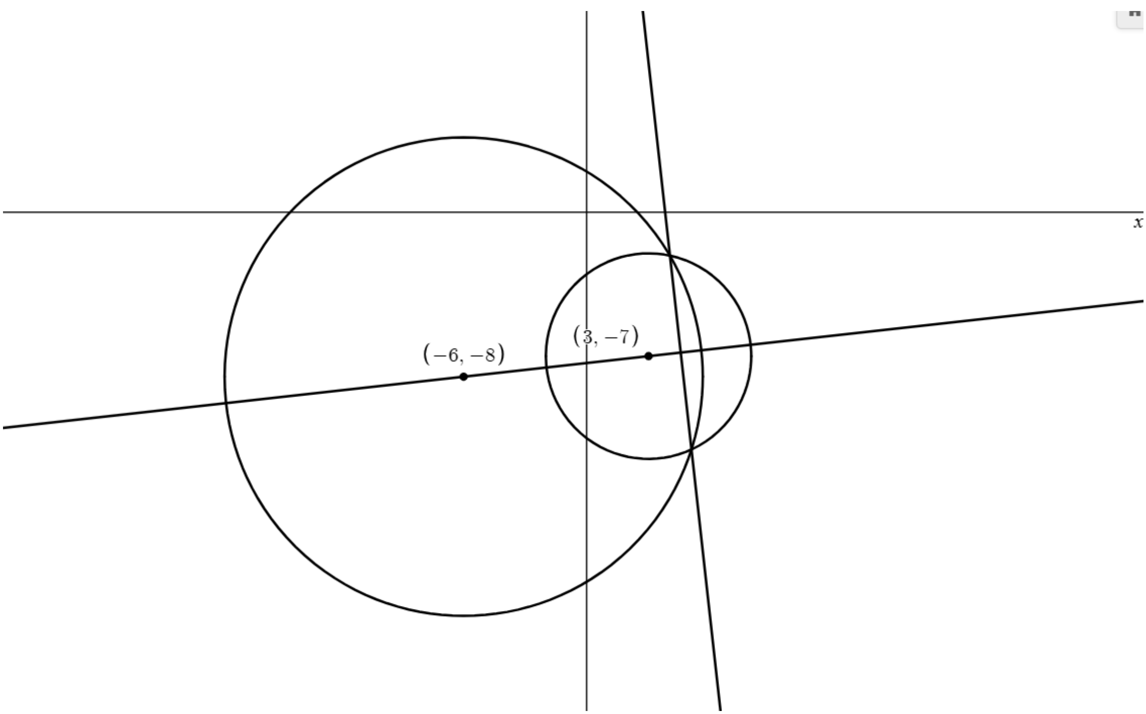
dM1: Correct strategy for one of the limits. E.g. adds or subtracts <u>their 5</u> to their distance between centres.

A1: Correct limits. There is no follow through but allow decimals to 3sf e.g. awrt 4.06 and awrt 14.1

A1: Correct answer with **exact values** using set notation.
Allow as shown in the main scheme but also allow equivalent set notation e.g.
$\{k : k \in \mathbb{R}, \sqrt{82}-5 < k < \sqrt{82}+5\}$, $\{k : \sqrt{82}-5 < k < \sqrt{82}+5\}$, $k \in (\sqrt{82}-5, \sqrt{82}+5)$
and allow "|" for ":" and allow the "k:" or "$k \in$" to be missing
e.g. $\{\sqrt{82}-5 < k < \sqrt{82}+5\}$ and $(\sqrt{82}-5, \sqrt{82}+5)$ are both acceptable.
But $\{k : k < \sqrt{82}+5\} \cup \{k : k > \sqrt{82}-5\}$ or $\{k : k < \sqrt{82}-5, k < \sqrt{82}+5\}$ score A0
Do **not** allow solutions not in set notation such as $\sqrt{82}-5 < k < \sqrt{82}+5$

Correct answers with no working should be sent to review.

Scenario for part (b) for reference:



Algebraic approach for part (b):

$$x^2+y^2-6x+14y+33=x^2+12x+36+y^2+16y+64-k^2$$

$$\Rightarrow 18x+2y+67-k^2=0 \Rightarrow y=\frac{k^2-67}{2}-9x$$

$$(x-3)^2+(y+7)^2=25 \Rightarrow x^2-6x+9+\left(\frac{k^2-67}{2}-9x+7\right)^2=25$$

$$\Rightarrow 82x^2+471x-9k^2x+\frac{k^4-106k^2+2745}{4}=0$$

When circles touch $b^2-4ac=0$

$$\Rightarrow \left(471-9k^2\right)^2-4\times 82\left(\frac{k^4-106k^2+2745}{4}\right)=0$$

$$\Rightarrow k^4-214k^2+3249=0$$

$$\Rightarrow \left(k^2-10k-57\right)\left(k^2+10k-57\right)=0$$

$$\Rightarrow k=5+\sqrt{82},\ 5-\sqrt{82},\ -5+\sqrt{82},\ -5-\sqrt{82}$$

$$k=\underline{5+\sqrt{82},\ -5+\sqrt{82}}$$

We will mark this as follows:

M1: This requires a valid strategy that:

- solves the 2 circle equations simultaneously to find *y* in terms of *x* and *k*, or *x* in terms of *y* and *k*
- substitutes for *y* or *x* into one of the circle equations to obtain an equation in *x* and *k* only, or *y* and *k* only,
- attempts $b^2-4ac=0$ or e.g. $b^2-4ac>0$ or equivalent to obtain an equation in *k* only. You do not need to look at the details of their algebra.

A1: Correct simplified 3TQ in k^2

dM1: Solves their 3TQ in k^2 by any correct method including a calculator to find *k*.

A1: Both correct values for *k* (exact or decimals as in the main scheme) (they may have extras which can be ignored)

A1: As main scheme (exact and in set notation)

Note that work such as

$$(x+6)^2+(y+8)^2=k^2 \Rightarrow x+6+y+8=k$$

is not a valid strategy as it greatly simplifies the problem and would generally score no marks.

Implicit differentiation approach for part (b):

$$x^2+y^2-6x+14y+33=0 \Rightarrow 2x+2y\frac{\mathrm{d}y}{\mathrm{d}x}-6+14\frac{\mathrm{d}y}{\mathrm{d}x}=0 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{6-2x}{2y+14}$$

$$(x+6)^2+(y+8)^2=k^2 \Rightarrow 2(x+6)+2(y+8)\frac{\mathrm{d}y}{\mathrm{d}x}=0 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{x+6}{y+8}$$

$$\frac{3-x}{y+7}=-\frac{x+6}{y+8} \Rightarrow (3-x)(y+8)=-(x+6)(y+7) \Rightarrow x=9y+66$$

$$(9y+66)^2+y^2-6(9y+66)+14y+33=0 \Rightarrow 82y^2+1148y+3993=0$$

$$(\text{or } 82x^2-492x-1287=0\text{ })$$

$$y=\frac{-574\pm5\sqrt{82}}{82} \Rightarrow x=\frac{246\pm45\sqrt{82}}{82}$$

$$\left(\frac{246+45\sqrt{82}}{82},\frac{-574+5\sqrt{82}}{82}\right) \to \left(\frac{246+45\sqrt{82}}{82}+6\right)^2+\left(\frac{-574+5\sqrt{82}}{82}+8\right)^2=k^2$$

$$\Rightarrow k^2=107+10\sqrt{82} \Rightarrow k=5+\sqrt{82}$$

$$\text{Then the same for } \left(\frac{246-45\sqrt{82}}{82},\frac{-574-5\sqrt{82}}{82}\right) \to k=-5+\sqrt{82}$$

We will mark this as follows:

M1: This requires a valid strategy that:

- differentiates the equations of both circles implicitly and equates the derivatives to obtain an equation connecting *y* and *x*. (Note that the equation connecting *y* and *x* is the common equation through the centres which can also be found from using the coordinates of the centres)
- substitutes for *x* or *y* into the equation for C_1 to obtain an equation in one variable

A1: Correct 3TQ in *y* or *x*

dM1: This requires:

- solves their 3TQ in *y* or *x* by any correct means including a calculator and finds at least one point of intersection
- substitutes this point into C_2 and proceeds to a value for *k*

A1: Correct values for *k* (exact or decimals as in the main scheme)

A1: As main scheme (exact and in set notation)