Question	Scheme	Marks	AOs
15(a)	$3(x+y)^2\left(1+\dfrac{dy}{dx}\right)=6x-3\dfrac{dy}{dx}$	M1 A1 A1	3.1a 1.1b 1.1b
	$\left(3(x+y)^2+3\right)\dfrac{dy}{dx}=6x-3(x+y)^2 \Rightarrow \dfrac{dy}{dx}=...$	M1	2.1
	$\dfrac{dy}{dx}=\dfrac{6x-3(x+y)^2}{3(x+y)^2+3}\ \left(\text{oe e.g. } \dfrac{2x-(x+y)^2}{(x+y)^2+1}\right)$	A1	1.1b
		(5)	
	Alternative – expands $(x+y)^3$ before differentiating $(x+y)^3=x^3+3x^2y+3xy^2+y^3$		
	$\Rightarrow 3x^2+3x^2\dfrac{dy}{dx}+6xy+6xy\dfrac{dy}{dx}+3y^2+3y^2\dfrac{dy}{dx}=6x-3\dfrac{dy}{dx}$	M1 A1 A1	3.1a 1.1b 1.1b
	$\left(3x^2+6xy+3y^2+3\right)\dfrac{dy}{dx}=6x-3x^2-6xy-3y^2 \Rightarrow \dfrac{dy}{dx}=...$	M1	2.1
	$\left(3x^2+6xy+3y^2+3\right)\dfrac{dy}{dx}=6x-3x^2-6xy-3y^2$ $\Rightarrow \dfrac{dy}{dx}=\dfrac{6x-3x^2-6xy-3y^2}{3x^2+6xy+3y^2+3}\ \left(\text{oe e.g. } \dfrac{2x-x^2-2xy-y^2}{x^2+2xy+y^2+1}\right)$	A1	1.1b

(a) Notes

(a) Some candidates have a spurious "$\dfrac{dy}{dx}=$" appearing as their intention to differentiate e.g.

$$\left(\frac{dy}{dx}=\right)3(x+y)^2\left(1+\frac{dy}{dx}\right)=6x-3\frac{dy}{dx}$$

This can be condoned for the first 3 marks in both versions.

Allow equivalent notation for the $\dfrac{dy}{dx}$ e.g. y'

M1: Award this mark for one of:

- $(x+y)^3 \to k(x+y)^2\left(\lambda+\dfrac{dy}{dx}\right)$ where λ is 1, x or 0 but condone missing brackets e.g.

  $3(x+y)^2 1+\dfrac{dy}{dx}$

- $3x^2-3y-2 \to ...x+...\dfrac{dy}{dx}$ but condone $3x^2-3y-2 \to ...x+...\dfrac{dy}{dx}-2$

A1: **Either** $3(x+y)^2\left(1+\dfrac{dy}{dx}\right)$ **or** $6x-3\dfrac{dy}{dx}$ oe

May be implied if e.g. they collect terms to one side initially.
Do not condone missing brackets unless they are implied by subsequent work.

A1: $3(x+y)^2\left(1+\dfrac{dy}{dx}\right)$ **and** $6x-3\dfrac{dy}{dx}$ (seen separately or equated)

If they collect terms to one side initially then the signs must be correct.

M1: A valid attempt to make $\frac{dy}{dx}$ the subject with exactly 2 **different** terms in $\frac{dy}{dx}$, one coming from the differentiation of $(x+y)^3$ and the other coming from the differentiation of "$-3y$"

Note that here, 2 **different** terms means terms such as $3\frac{dy}{dx}$ and $3(x+y)^2\frac{dy}{dx}$ and not e.g. $3\frac{dy}{dx}$ and $-8\frac{dy}{dx}$

Look for $(...\pm...)\frac{dy}{dx}=...\Rightarrow\frac{dy}{dx}=...$ which may be implied by their working.

Condone slips provided the intention is clear.

For those candidates who had a spurious $\frac{dy}{dx}=...$ at the start, they may incorporate this in their rearrangement in which case they will have 3 terms in $\frac{dy}{dx}$ and so score M0.

If they ignore it, then this mark is available for the condition as described above.

Note that from $3(x+y)^2\left(1+\frac{dy}{dx}\right)=6x-3\frac{dy}{dx}$, candidates may expand the brackets before rearranging, in which case they would need 4 **different** $\frac{dy}{dx}$ terms coming from the appropriate places.

Note that the different $\frac{dy}{dx}$ terms do not have to be correct as long as the above conditions are satisfied.

A1: Fully correct expression for $\frac{dy}{dx}$. Allow any equivalent correct forms.

Apply isw as soon as a correct expression is seen.

(a) alternative by expanding:

M1: Award this mark for one of:

- $3x^2-3y-2\to...x+...\frac{dy}{dx}$ but condone $3x^2-3y-2\to...x+...\frac{dy}{dx}-2$
- Expanding $(x+y)^3$ to obtain either an x^2y term or an xy^2 term and then uses the product rule to obtain $...x^2y\to...x^2\frac{dy}{dx}+...xy$ or $...xy^2\to...xy\frac{dy}{dx}+...y^2$

A1: **Either** $3x^2+3x^2\frac{dy}{dx}+6xy+6xy\frac{dy}{dx}+3y^2+3y^2\frac{dy}{dx}$ **or** $6x-3\frac{dy}{dx}$.

May be implied if e.g. they collect terms to one side initially.

A1: $3x^2+3x^2\frac{dy}{dx}+6xy+6xy\frac{dy}{dx}+3y^2+3y^2\frac{dy}{dx}$ **and** $6x-3\frac{dy}{dx}$ oe. (seen separately or equated) If they collect terms to one side initially then the signs must be correct.

M1: A valid attempt to make $\frac{dy}{dx}$ the subject with exactly 4 **different** terms in $\frac{dy}{dx}$, 3 coming from the differentiation of $(x+y)^3$ and the other coming from the differentiation of "$-3y$"

Note that here, 4 **different** terms means terms such as $x^2\frac{dy}{dx}$ and $6xy\frac{dy}{dx}$ and not e.g.

$3\frac{dy}{dx}$ and $-8\frac{dy}{dx}$

Look for $(...\pm...\pm...\pm...)\frac{dy}{dx} = ... \Rightarrow \frac{dy}{dx} = ...$ which may be implied by their working.

Condone slips provided the intention is clear.

For those candidates who had a spurious $\frac{dy}{dx} = ...$ at the start, they may incorporate this in their rearrangement in which case they will have 5 terms in $\frac{dy}{dx}$ and so score M0.

If they ignore it, then this mark is available for the condition as described above.

Note that the different $\frac{dy}{dx}$ terms do not have to be correct as long as the above conditions are satisfied. E.g. if they have an incorrect term such as $6x\frac{dy}{dx}$, this mark is still available.

A1: Fully correct expression for $\frac{dy}{dx}$. Allow any equivalent correct forms.

Condone e.g. $3x2y$ for $6xy$.

Apply isw as soon as a correct expression is seen.

Alternative making *y* the subject in (a):

$$(x+y)^3 = 3x^2 - 3y - 2$$

$$x + y = (3x^2 - 3y - 2)^{\frac{1}{3}} \Rightarrow y = (3x^2 - 3y - 2)^{\frac{1}{3}} - x$$

$$\frac{dy}{dx} = \frac{1}{3}(3x^2 - 3y - 2)^{-\frac{2}{3}}\left(6x - 3\frac{dy}{dx}\right) - 1$$

$$\frac{dy}{dx}\left(1 + (3x^2 - 3y - 2)^{-\frac{2}{3}}\right) = 2x(3x^2 - 3y - 2)^{-\frac{2}{3}} - 1$$

$$\frac{dy}{dx} = \frac{2x(3x^2 - 3y - 2)^{-\frac{2}{3}} - 1}{1 + (3x^2 - 3y - 2)^{-\frac{2}{3}}}$$

Score as follows:

M1: Cube roots both sides and makes $x + y$ or y the subject then award for

- $(3x^2 - 3y - 2)^{\frac{1}{3}} \to ...(3x^2 - 3y - 2)^{-\frac{2}{3}}$ or
- $3x^2 - 3y - 2 \to ...x + ...\frac{dy}{dx}$ but condone $3x^2 - 3y - 2 \to ...x + ...\frac{dy}{dx} - 2$

A1: For the $\frac{1}{3}(3x^2 - 3y - 2)^{-\frac{2}{3}}$ **or** $6x - 3\frac{dy}{dx}$

A1: Fully correct

M1: A valid attempt to make $\frac{dy}{dx}$ the subject with exactly 2 **different** terms in $\frac{dy}{dx}$

A1: Correct expression

Using partial derivatives in (a):

$$(x+y)^3 = 3x^2 - 3y - 2 \rightarrow \text{f}(x,y) = (x+y)^3 - 3x^2 + 3y + 2$$

$$\frac{\partial \text{f}}{\partial x} = 3(x+y)^2 - 6x \qquad \frac{\partial \text{f}}{\partial y} = 3(x+y)^2 + 3$$

$$\frac{\text{d}y}{\text{d}x} = -\frac{\partial \text{f}}{\partial x} \div \frac{\partial \text{f}}{\partial y} = \frac{6x - 3(x+y)^2}{3(x+y)^2 + 3}$$

or

$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3 = 3x^2 - 3y - 2$$

$$\text{f}(x,y) = x^3 + 3x^2y + 3xy^2 + y^3 - 3x^2 + 3y + 2$$

$$\frac{\partial \text{f}}{\partial x} = 3x^2 + 6xy + 3y^2 - 6x \qquad \frac{\partial \text{f}}{\partial y} = 3x^2 + 6xy + 3y^2 + 3$$

$$\frac{\text{d}y}{\text{d}x} = -\frac{\partial \text{f}}{\partial x} \div \frac{\partial \text{f}}{\partial y} = \frac{-3x^2 - 6xy - 3y^2 + 6x}{3x^2 + 6xy + 3y^2 + 3}$$

Score as follows:

M1: Correct structure for either partial derivative:

Doesn't expand: $\frac{\partial \text{f}}{\partial x} = \ldots(x+y)^2 + \ldots x$ or $\frac{\partial \text{f}}{\partial y} = \ldots(x+y)^2 + \ldots$

or

Expands: $\frac{\partial \text{f}}{\partial x} = \ldots x^2 + \ldots xy + \ldots y^2 + \ldots x$ or $\frac{\partial \text{f}}{\partial y} = \ldots x^2 + \ldots xy + \ldots y^2 + \ldots$

Where "…" are non-zero constants

A1: Correct $\frac{\partial \text{f}}{\partial x}$ **or** correct $\frac{\partial \text{f}}{\partial y}$

A1: Correct $\frac{\partial \text{f}}{\partial x}$ **and** correct $\frac{\partial \text{f}}{\partial y}$

M1: Attempts $\frac{\text{d}y}{\text{d}x} = -\frac{\partial \text{f}}{\partial x} \div \frac{\partial \text{f}}{\partial y}$

A1: Correct expression

(b)	$\frac{dy}{dx}=\frac{6(1)-3(0+1)^2}{3(0+1)^2+3}=\frac{1}{2}$ or e.g. $\frac{dy}{dx}=\frac{6(1)-3(1)^2-6(1)(0)-3(0)^2}{3(1)^2+6(1)(0)+3(0)^2+3}=\frac{1}{2}$ $\Rightarrow y-0=-2(x-1)$ **or** $\Rightarrow y=-2x+c\Rightarrow 0=-2+c\Rightarrow c=...$	M1	2.1
	$y=-2x+2$*	A1*	1.1b
		(2)	

(b) Notes

(b) **Note that the gradient of $\frac{1}{2}$ could have been deduced from the given equation so you will need to check their solution carefully.**

M1: Substitutes $x = 1$ and $y = 0$ into their $\frac{dy}{dx}$ to obtain the tangent gradient **and** then uses the negative reciprocal and $x = 1$ and $y = 0$ in a correct straight line method to obtain the normal equation with $x = 1$ and $y = 0$ correctly placed.
Note that when finding the normal gradient, they may find the negative reciprocal of their expression from part (a) and then substitute $x = 1$ and $y = 0$ which is fine.
If using $y = mx + c$ they must proceed as far as finding a value for c.
If no substitution of $x = 1$ and $y = 0$ into their $\frac{dy}{dx}$ is seen you will need to check their value. If they **just** state a value for $\frac{dy}{dx}$ then it must follow their $\frac{dy}{dx}$ with $x = 1$and $y = 0$

A1*: Correct equation with no errors **following a correct** $\frac{dy}{dx}$ from part (a) (unless they start again which is unlikely)
Be aware that some incorrect expressions for $\frac{dy}{dx}$ from part (a) may fortuitously give $\frac{dy}{dx}=\frac{1}{2}$ and would generally score A0
In general A1* must follow the final A1 in (a) or correct differentiation in (a)

(c)	$y=-2x+2 \Rightarrow (x-2x+2)^3 = 3x^2-3(-2x+2)-2$ **or** $x=\dfrac{2-y}{2} \Rightarrow \left(\dfrac{2-y}{2}+y\right)^3 = 3\left(\dfrac{2-y}{2}\right)^2 - 3y-2$	M1	1.1b
	$x^3-3x^2+18x-16=0$ **or** $y^3+60y=0$	A1	1.1b
	$\Rightarrow (x-1)(x^2-2x+16)=0$ (x = 1 is known) or $\Rightarrow y(y^2+60)=0$ (y = 0 is known)	dM1	2.1
	For $x^2-2x+16=0,\ \ b^2-4ac=4-4\times1\times16$ or For $y^2+60=0,\ \ y^2\neq-60$	ddM1	2.1
	As $b^2-4ac<0$ or as $y^2\neq-60$ there are no other real roots and so the normal does not meet C again.	A1	2.4
		(5)	

(c) Notes

(c)

M1: Uses the equation from part (a) and substitutes $y=\pm2x\pm2$ or $x=\dfrac{\pm2\pm y}{2}$ to obtain an equation in one variable (usually x) (not necessarily a cubic equation). Allow slips in rearranging to obtain x in terms of y (or y in terms of x) as long as the intention is clear.

A1: Correct cubic equation with terms collected and "= 0" seen or implied.
Note that both $-x^3+3x^2-18x+16=0$ and $-y^3-60y=0$ are correct equations.

To access any of the following marks, candidates must attempt to use either the factor of (x – 1) with their cubic in x or the factor of y in their cubic in y to obtain a quadratic expression in x or y.
Attempts that just use a calculator to solve the cubic equation score no more marks in this part.

dM1: Uses the fact that (x – 1) or y is a factor in an attempt to establish the quadratic factor.
For the cubic in x, it must be of the form $ax^3+bx^2+cx+d=0 \quad a,b,c,d\neq0$
For the cubic in y, it must be of the form $ay^3+by=0 \quad a,b\neq0$
For the cubic in x, the attempt at the quadratic factor using (x – 1) may be via inspection or e.g. long division to obtain a 3 term quadratic expression. There may or may not be a remainder but they must obtain 3 terms.
For the cubic in y, they would need to take out a factor of y (or divide through by y) to obtain a factor of the form $k(y^2+\alpha)$

ddM1: This mark requires:

- a correct cubic equation in x or y
- the correct quadratic factor or a multiple of it e.g. $k\left(x^2-2x+16\right)$ or $k\left(y^2+60\right)$
- an attempt to show that the quadratic factor has no real roots

For the quadratic in *x* this could be:

Attempts discriminant: e.g. $b^2-4ac=4-4\times1\times16$ (may be embedded in the quadratic formula)

Attempts to complete the square: e.g. $x^2-2x+16=(x-1)^2-1+16$

Uses calculus to find the turning point: e.g. $\frac{\mathrm{d}\left(x^2-2x+16\right)}{\mathrm{d}x}=2x-2=0\Rightarrow x=1\Rightarrow y=...$

Attempts to solve: e.g. $x^2-2x+16=0\Rightarrow x=\frac{2\pm\sqrt{4-4\times16}}{2}$ or from a calculator $x=1\pm\sqrt{15}\mathrm{i}$

For the quadratic in *y* this is likely to be:

Attempts to solve: e.g. $y^2+60=0\Rightarrow y^2=-60\Rightarrow ...$

A1: Fully correct argument that requires:

- fully correct work
- a justification depending on their strategy
- a conclusion depending on their strategy

Via discriminant: $4-4\times1\times16<0$ so no real roots so they do not meet again

Via completing the square: $\rightarrow(x-1)^2+15$ which has a minimum value of 15 so no real roots so they do not meet again.

Via calculus: $x=1\Rightarrow y=15$ is the minimum so no real roots so they do not meet again.

Via solving: $x=1\pm\sqrt{15}\mathrm{i}$ or e.g. $x=1\pm3.87\mathrm{i}$ or e.g. $x=1\pm\sqrt{-15}$ so math error so they do not meet again.

For y it is likely to be more straightforward e.g. $y^2\neq-60$ which cannot be solved so they do not meet again.

Allow equivalent statements for "they do not meet again" e.g. so they only meet once.

(But do **not** condone incorrect statements such as "therefore P does not meet C again")

A minimum justification could be:

$$x^2-2x+16=0\rightarrow b^2-4ac=(-2)^2-4\times1\times16 \quad \textbf{ddM1}$$

$4-4\times1\times16<0$ so no more roots so no more intersections **A1**

Do not allow e.g.

"$x^2-2x+16=0$ gives a math error so they do not meet again"

as there has been no attempt to show why the "math error" occurs – this scores M0A0

Alternative to (c) by showing the cubic is strictly increasing (or decreasing):

M1A1: As in the main scheme then

$$\mathrm{f}(x)=x^3-3x^2+18x-16\Rightarrow \mathrm{f}'(x)=3x^2-6x+18$$

$$3x^2-6x+18=3\left(x^2-2x+6\right)=3(x-1)^2+15$$

$$3(x-1)^2+15>0 \text{ so f}(x) \text{ is an increasing function}$$

Hence there can only be one intersection (at x = 1) so the normal and curve do not intersect again.

dM1: Differentiates their cubic of the form $ax^3+bx^2+cx+d=0 \quad a,b,c,d\neq 0$ to obtain a 3 term quadratic expression with only coefficient errors on the non-constant terms.

ddM1: This mark requires:

- a correct cubic equation in x
- the correct derivative or a multiple of it
- an attempt to show that the quadratic expression is always positive (or negative)

A1: Fully correct concluding argument e.g. that as the derivative is always positive (or always negative) the function is strictly increasing (or decreasing) and therefore there can only be one intersection (at x = 1) so the normal and curve do not meet again.

(12 marks)