

Question	Scheme	Marks	AOs
6(a)	Attempts $\cos^2(2x) \approx \left(1 - \frac{(2x)^2}{2}\right)^2 = \dots(1 - 4x^2 + 4x^4)$	M1	1.1b
	$1 - \cos^2(2x) \approx 1 - (1 - 4x^2 + 4x^4) = 4x^2 - 4x^4 *$	A1*	1.1b
		(2)	
(b)	$\frac{1 - \cos^2(2x)}{\sin\left(\frac{x}{3}\right)\tan\left(\frac{x}{2}\right)} \approx \frac{4x^2 - 4x^4}{\left(\frac{x}{3}\right)\left(\frac{x}{2}\right)}$	M1	1.1b
	$= \frac{6}{x^2}(4x^2 - 4x^4) = 24 - 24x^2$	A1	2.1
		(2)	
(c)	24	B1ft	2.2a
	If x is (very) small, then any terms in x^2 are negligible.	dB1ft	2.4
		(2)	

(6 marks)

Notes

<p>(a)</p> <p>M1: Attempts to use $\cos \theta \approx 1 - \frac{\theta^2}{2}$ with θ, 2θ, x or $2x$ and squares the resulting expression.</p> <p>Condone poor squaring, e.g., $(1 - 2x^2)^2 = 1 - 4x^4$ or missing brackets $\left(1 - \frac{2x^2}{2}\right)^2 = (1 - x^2)^2 = \dots$</p> <p>but not e.g. $\cos 2x \approx 2\left(1 - \frac{x^2}{2}\right)$</p> <p>May be implied by e.g. $1 - (1 - 2x^2)^2 = 2x^2(2 - 2x^2)$ from difference of two squares.</p> <p>A1*: Correct proof with an intermediate line such as $1 - (1 - 4x^2 + 4x^4)$</p> <p>Do not be concerned if the LHS does not appear and ignore any spurious $= 0$ in their work.</p> <p>There should be no obvious incorrect statements in the proof e.g. $1 - (1 + 4x^2 - 4x^4)$</p> <p>Do not condone incorrect work including invisible brackets such as $1 - 1 - 4x^2 + 4x^4$ or $\left(1 - \frac{2x^2}{2}\right)$ but condone a missing trailing bracket e.g. $1 - (1 - 4x^2 + 4x^4$</p> <p>Condone an attempt that starts in stages, which may not deal with the full expression e.g., $1 - \left(1 - \frac{4x^2}{2}\right) \rightarrow 1 - (1 - 2x^2)^2 = \dots$ has a missing square on the first bracket but is recovered in the next stage/step.</p> <p>The final line should be in terms of x but condone a slip to e.g. θ in the workings of the proof.</p> <p>If they complete the proof using θ they must revert to x to score this mark.</p> <p>Special Cases: In each of these cases below, please send to review.</p> <p>You may see attempts that use e.g.</p> <ul style="list-style-type: none"> $\cos 2x = \pm 2 \cos^2 x \pm 1$ or $\cos^2 2x = \pm \frac{1}{2} \pm \frac{1}{2} \cos 4x$ $\cos 2x = \pm 1 \pm 2 \sin^2 x$ followed by the small angle approximation for $\sin x$ Maclaurin expansions
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(b)

M1: Uses the **given answer** to part (a) and writes $\sin\left(\frac{x}{3}\right)$ **and** $\tan\left(\frac{x}{2}\right)$ using correct small angle approximations. Only allow misreads that are clearly misreads e.g. they cannot replace $\tan\left(\frac{x}{2}\right)$ with $\frac{x}{3}$ unless $\tan\left(\frac{x}{3}\right)$ is seen first. In such cases they will lose the A mark in (b) but both of the marks in (c) are available. Condone mixed variables for this mark.

A1: $24 - 24x^2$ but condone e.g. $24 + -24x^2$ or e.g. $a = 24$ and $b = -24$
Condone recovery of missing/invisible brackets but the work must otherwise be correct.
Do not condone mixed variables being recovered unless explicitly replaced with x before cancelling x^2 .

No marks are scored in (b) for using the Maclaurin expansions for $\sin\left(\frac{x}{3}\right)$ and/or $\tan\left(\frac{x}{2}\right)$

(c)

B1ft: 24 but this must follow from the non-zero constant term in their answer to (b).
Do not allow e.g. $x = 24$

Allow follow through on their non-zero constant term from a polynomial in x .

dB1ft: Suitable reason given but it should refer to their x^2 term in some way (and any additional terms if they have any). Ignore spurious remarks e.g. “if $x < 1$ ” unless contradictory.
Dependent on the previous B1ft mark.

Some acceptable examples:

- “ $24x^2 \rightarrow 0$ or $x^2 \rightarrow 0$ ”
- “We can ignore the x^2 term”
- “ x^2 is much smaller than 24”
- As $x \rightarrow 0$ their $a + bx^2 \rightarrow a$ (condone as $x \rightarrow 0$, bx^2 “becomes” 0)
- $\lim_{x \rightarrow 0} a + bx^2 = (a + b(0)^2) = a$
- Since x is very small, $24 - 24(0)^2 = 24$ (and condone if the squared is missing).

They cannot *just* substitute in 0 or a very small value for x to score the mark for the reason.

A reason such as “the answer rounds to 24” is not acceptable.

There must be some justification, either “as $x \rightarrow 0$ ” or “since x is very small”

so e.g. “ $24 - 24(0)^2 = 24$ on its own scores B1ft dB0ft

This mark may be scored from any polynomial in x that includes at least a constant term and an x^2 term.

They cannot go back to e.g. $4x^4$ or $\frac{4x^4}{\frac{1}{6}x^2}$ and say this is negligible or $\rightarrow 0$ as they haven’t

dealt with the x in the denominator.