Question	Scheme	Marks	AOs
7	Any one of $A=3,\ B=-2,\ C=4,\ D=-5$ correct from correct work	B1	2.2a
	Finds the value of B, C or D using a valid method e.g. $3x^3-8x^2-6x-11=(Ax+B)(x+1)(x-3)+C(x-3)+D(x+1)$ **and** substitutes $x=-1\Rightarrow\ldots=\ldots C\Rightarrow C=\ldots$ **or** substitutes $x=3\Rightarrow\ldots=\ldots D\Rightarrow D=\ldots$	M1	1.1b
	Valid method for finding all four constants. e.g. Deduces A = 3, substitutes x = 3 leading to D = … **and** substitutes x = –1 leading to C = … **and** substitutes e.g. x = 0 leading to B = …	M1 (A1 on EPEN)	1.1b
	$A=3,\ \ B=-2,\ C=4$ and $D=-5$	A1	2.1
		(4)	

(4 marks)

Notes

Note: Condone different uses of 'A', 'B' etc for the M marks provided the intention is clear but values must be correctly attributed to each letter or implied (if embedded in a final expression) for the A mark.

B1: Any one of $A=3,\ B=-2,\ C=4,\ D=-5$ correct (usually A = 3 is deduced) from correct work. May be implied by $3x$ seen. Condone $A = 3x$ for this mark.

M1: This mark is for finding the value of B, C or D, using a valid method, which may be in one of the following ways:

- Attempts algebraic division of $(3x^3-8x^2-6x-11)$ by their expanded $(x+1)(x-3)=("x^2-2x-3")$ **or** separately by both $(x+1)$ **and** $(x-3)$ as far as $\mathbf{3x\pm 2}$ Ignore any remainder for this mark. For reference, the correct remainder is $-x-17$

- Algebraic division might be seen as $3x(x^2-2x-3)\pm 2(x^2-2x-3)+"9x-4x-6-6x-11"$ o.e. and scores the B1 for the $3x$ in the correct place and M1 for both the $3x$ and the ± 2 in the correct places.

  In the next two bullets, they should achieve $3x^3-8x^2-6x-11=(Ax+B)(x+1)(x-3)+C(x-3)+D(x+1)$ o.e. but condone small slips e.g. sign errors or a slip on a constant. The bracketed expressions should be in the correct places and Ax and B should both be multiplied by $(x+1)(x-3)$.
  Note that $3x^3-8x^2-6x-11=C(x-3)+D(x+1)$ without sight of the identity above (condoning the previously mentioned slips) is an incorrect method and scores M0.

- Attempts to multiply up by $(x+1)(x-3)$ and substitutes one of $x=-1$ **or** $x=3$ leading to $C=...$ **or** $D=...$ (you do not need to check the accuracy of their substitution)

- **or** attempts to multiply up by $(x+1)(x-3)$, expands and compares coefficients leading to a value for B, C or D. Do not be concerned about how they expand the brackets.

- Attempts at the cover up method may be seen. If unsure then send to review.

M1: For finding values of all four constants via a valid method.
Not dependent on the previous method mark so B1M0M1A0 is possible.

- If attempting algebraic division, they must have a quotient of $3x \pm k$ for constant k ($3x \pm 2$ is **not** required for this mark) **and** they must go on to deal with their remainder using a correct method, i.e., $"-x-17" = C(x-3)+D(x+1)$ followed by either comparing coefficients or substitution of appropriate values. You do not need to check the accuracy of their substitution.

- For those attempting repeated algebraic division, the values of C and D must come from a correct method and so cannot be equal to e.g. the remainders from successive division by $(x+1)$ and $(x-3)$ or vice versa. If, e.g., they divide by $(x+1)$ (to get remainder R_1) and then $(x-3)$ (to get remainder R_2) they should get $3x \pm k + \frac{R_1}{(x+1)(x-3)} + \frac{R_2}{(x-3)}$. They can then either combine the two fraction terms before using partial fractions or they can use partial fractions on $\frac{R_1}{(x+1)(x-3)}$ before combining their two terms in $\frac{1}{(x-3)}$.

- If using $3x^3 - 8x^2 - 6x - 11 = (Ax+B)(x+1)(x-3) + C(x-3) + D(x+1)$ o.e. they may use a combination of substitution and comparing coefficients, but the method must be valid. Do not be concerned about how they expand the brackets, and you do not need to check the accuracy of their substitution.

- If using substitution only they are likely to use $x = -1$, 3 and 0 but the final value for x could be anything (and in fact they could use three/four different values for x). You do not need to check the accuracy of their substitution.

- If attempting to compare coefficients, do not be concerned about how they expand the brackets.

A1: Requires:

- all of $A=3$, $B=-2$, $C=4$ and $D=-5$ which may be stated separately in their work ($A = 3x$ that is not recovered would score final A0).
- **or** $3x - 2 + \frac{4}{x+1} - \frac{5}{x-3}$ seen. Condone $3x + -2 + \frac{4}{x+1} + \frac{-5}{x-3}$

There is no requirement to write out the form given in the question once correct values for A, B, C and D have been seen.

ISW after the four correct values (e.g. if they swap C and D in the fractions so $A=3,\ B=-2,\ C=4,\ D=-5$ followed by $3x-2-\frac{5}{x+1}+\frac{4}{x-3}$ scores full marks).

Repeated Algebraic Division

An example for repeated algebraic division. Note that the repeated division could be carried out in the other order – i.e. dividing by $(x - 3)$ first followed by $(x + 1)$.

$$
\begin{array}{r|l}
 & 3x^2 - 11x + 5 \\
\hline
x + 1 & 3x^3 - 8x^2 - 6x - 11 \\
 & 3x^3 + 3x^2 \\
\hline
 & -11x^2 - 6x \\
 & -11x^2 - 11x \\
\hline
 & 5x - 11 \\
 & 5x + 5 \\
\hline
 & -16
\end{array}
$$

Followed by

$$
\begin{array}{r|l}
 & 3x - 2 \\
\hline
x - 3 & 3x^2 - 11x + 5 \\
 & 3x^2 - 9x \\
\hline
 & -2x + 5 \\
 & -2x + 6 \\
\hline
 & -1
\end{array}
$$

- **B1** Scored for either '3' **or** '–2' embedded in the quotient.
- **1st M1** Scored for achieving $3x \pm 2$

$$\frac{3x^3 - 8x^2 - 6x - 11}{(x + 1)(x - 3)} = 3x - 2 + \frac{-16}{(x + 1)(x - 3)} + \frac{-1}{(x - 3)}$$

$$\frac{3x^3 - 8x^2 - 6x - 11}{(x + 1)(x - 3)} = 3x - 2 + \frac{-16}{(x + 1)(x - 3)} + \frac{-(x + 1)}{(x + 1)(x - 3)}$$

- Correctly deals with both remainders from the repeated division.

$$\frac{3x^3 - 8x^2 - 6x - 11}{(x + 1)(x - 3)} = 3x - 2 + \frac{-x - 17}{(x + 1)(x - 3)}$$

$$-x - 17 = C(x - 3) + D(x + 1)$$

$$x = 3 \Rightarrow D = -5 \text{ and } x = -1 \Rightarrow C = 4$$

$$\boxed{A = 3,\ B = -2,\ C = 4.\ D = -5}$$

- Sets up an identity in x and C and D using partial fractions.
- **2nd M1** Finds the values of all four constants via a correct method (in this case by substitution of appropriate values).
- **A1** $A = 3$, $B = -2$. $C = 4$ and $D = -5$

$$\frac{3x^3 - 8x^2 - 6x - 11}{(x + 1)(x - 3)} = 3x - 2 + \frac{F}{(x + 1)} + \frac{G}{(x - 3)} + \frac{-1}{(x - 3)}$$

$$-16 = G(x + 1) + F(x - 3)$$

$$x = -1 \Rightarrow F = 4,\ x = 3 \Rightarrow G = -4$$

$$\frac{3x^3 - 8x^2 - 6x - 11}{(x + 1)(x - 3)} = 3x - 2 + \frac{4}{(x + 1)} + \frac{-4}{(x - 3)} + \frac{-1}{(x - 3)}$$

$$\boxed{\frac{3x^3 - 8x^2 - 6x - 11}{(x + 1)(x - 3)} = 3x - 2 + \frac{4}{(x + 1)} - \frac{5}{(x - 3)}}$$

- Alternatively, splits the first fraction using partial fractions and sets this equal to their remainder divided by $(x + 1)(x - 3)$ and multiplies up correctly.
- **2nd M1** Finds the values of all four constants via a valid method.
- **A1** $A = 3$, $B = -2$, $C = 4$ and $D = -5$ seen embedded in a final expression.