Question	Scheme	Marks	AOs
10(a)	$\frac{dV}{dt}=0.45$ or $\frac{dV}{dt}=\pm 0.3V$	M1	3.1b
	$\frac{dV}{dt}=0.45-\frac{3}{10}V$ $20\frac{dV}{dt}=9-6V$ *	A1*	2.1
		(2)	
(b)	e.g. $\frac{1}{9-6V}\frac{dV}{dt}=\frac{1}{20}\rightarrow\int\frac{1}{9-6V}dV=\int\frac{1}{20}dt$	B1	1.1b
	$\frac{1}{9-6V}\rightarrow\ldots\ln\lvert 9-6V\rvert$	M1	1.1b
	$-\frac{1}{6}\ln\lvert 9-6V\rvert=\frac{t}{20}\quad(+c)$	A1	1.1b
	$-\frac{1}{6}\ln\lvert 9-6V\rvert=\frac{t}{20}+c$ $9-6V=Ae^{-\frac{3t}{10}}$ $t=0,\ V=0.25\Rightarrow A=(7.5)$ $-\frac{1}{6}\ln\lvert 9-6V\rvert=\frac{t}{20}+c$ $t=0,\ V=0.25\Rightarrow c=\left(-\frac{1}{6}\ln 7.5\right)$	dM1	3.1a
	$9-6V=7.5e^{-\frac{3t}{10}}$ $V=\frac{3}{2}-\frac{5}{4}e^{-\frac{3t}{10}}$ $\frac{1}{6}\ln 7.5-\frac{1}{6}\ln\lvert 9-6V\rvert=\frac{t}{20}$ $\ln\frac{7.5}{9-6V}=0.3t$ $9-6V=7.5e^{-0.3t}$ $V=\frac{3}{2}-\frac{5}{4}e^{-0.3t}$	A1	2.1
		(5)	

Notes

(a) **Marks for part (a) may not be scored in part (b)**

M1: Either $\frac{dV}{dt}=0.45$ or $\frac{dV}{dt}=\pm 0.3V$ o.e. e.g. $\frac{dV}{dt}=\frac{9}{20}$ or $\frac{dV}{dt}=\pm\frac{3}{10}V$ seen or implied by e.g. $\frac{dV}{dt}=0.45-\frac{3}{10}V$ (but not implied by just stating the given answer). Condone use of $\dot{V}$

It may be seen as part of their $\frac{dV}{dt}$ e.g. $\frac{dV}{dt}=0.45+V+0.3V$ scores M1A0*

Condone e.g. change in volume = (inflow – outflow =) 0.45 – 0.3V for this mark.

A1*: Achieves $20\frac{dV}{dt}=9-6V$ with no errors, following $\frac{dV}{dt}=0.45-\frac{3}{10}V$ o.e. (including the $\frac{dV}{dt}$ or $\dot{V}$ but note that it must be $\frac{dV}{dt}$ in the final line and not $\dot{V}$).

change in volume = 0.45 – 0.3V $\rightarrow 20\frac{dV}{dt}=9-6V$ scores M1A0*.

Ignore any units used in their working for both marks.

(b)

B1: Separates the variables correctly, e.g., $\int\frac{1}{9-6V}dV=\int\frac{1}{20}dt$ or $\int\frac{20}{9-6V}dV=\int\{1\}dt$ o.e.

The integral symbol and/or dV and/or dt may be implied if they go on to integrate **both** sides to the correct form $\ldots\ln\lvert\alpha(9-6V)\rvert=\ldots t\quad(+c)$ with or without the modulus brackets.

M1:	Attempts to integrate the reciprocal term $\frac{\beta}{9-6V} \to \ldots \ln\lvert 9-6V\rvert$ or $\to \ldots \ln\lvert \alpha(6V-9)\rvert$ for some constant β (and α if used). Condone e.g. $\frac{20}{9-6V} \to \ldots \ln 9-6V$ or $\to \ldots \ln 6V-9$
A1:	Correct integration for both sides. They do not need the + c for this mark. Note scoring this mark implies the earlier B1 (unless it is a verification attempt – see SC). Note that e.g. $-\frac{1}{6}\ln\lvert 3-2V\rvert = \frac{t}{20}\ (+c)$ or $-\frac{10}{3}\ln\lvert 2V-3\rvert = t\ (+c)$ are also correct. $-\frac{1}{6}\ln(9-6V) = \frac{t}{20}\ (+c)$ is also correct. Condone log being used in place of ln.
dM1:	Requires constant of integration now. Substitutes (or states) $t = 0$ and $V = 0.25$ and finds a value for c, which may be "A" = e^c if they rearrange first to eliminate ln terms. Dependent on the previous method mark. Do not be concerned about their processing to find c or "A" = e^c and does not need to be exact.
A1:	Achieves the required form e.g. $V = \frac{3}{2} - \frac{5}{4}e^{-\frac{3t}{10}}$ with no errors and clear working. Allow equivalent fractions or decimals e.g. $V = 1.5 - 1.25e^{-\frac{6}{20}t}$
SC:	**Attempts by verification may score maximum B0M1A1dM1A0 – see below.**
Alt:	**Use of an integrating factor – see below.**
(b)	**Special Case: Attempts by verification may score maximum B0M1A1dM1A0**
B0:	This mark may not be scored via this approach.
M1:	Differentiates $V = P - Qe^{-kt}$ to the form $\frac{dV}{dt} = \alpha e^{-kt}$ where α is a constant (note it should be $\frac{dV}{dt} = Qke^{-kt}$) **and** substitutes both this and $V = P - Qe^{-kt}$ into $20\frac{dV}{dt} = 9-6V$ **and** deduces a value for P **or** k by comparing coefficients.
A1:	Correct values for both P **and** k.
dM1:	Substitutes (or states) $t = 0$ and $V = 0.25$ and finds a value for Q. Requires a value for P to have been found using the above approach.
A0:	This mark may not be scored via this approach.
	Alternative: Using Integrating Factor (Further Maths)
B1:	Deduces the correct integrating factor for the equation, $e^{0.3t}$ This should come from $\frac{dV}{dt} + 0.3V = 0.45 \Rightarrow \text{I.F.} = e^{\int 0.3\,dt} = e^{0.3t}$ May be implied by sight of $\frac{d(Ve^{0.3t})}{dt} = \ldots$
M1:	Fully multiplies through by their integrating factor and integrates both sides. Score for $Ve^{kt} = \int \ldots e^{kt}dt = \ldots e^{kt}$ Condone missing dt
A1:	Correct integration $Ve^{0.3t} = \int 0.45e^{0.3t}dt = \frac{3}{2}e^{0.3t}\ (+c)$
dM1:	As main scheme.
A1:	As main scheme.

Question	Scheme	Marks	AOs
10(c)	**Examples:** (1) $\frac{dV}{dt}=0 \Rightarrow V=(1.5)$ (or e.g. max V is 1.5) (2) As $t\to\infty$, $e^{-"0.3"t}\to 0$ (or $V\to$ "1.5") (3) Flow in = flow out at max V so $0.3V=0.45\Rightarrow V=1.5$ (4) As $e^{-"0.3"t}>0$, $V<"1.5"$ (5) When $V>1.5$, $\frac{dV}{dt}<0$ (6) $V=2\Rightarrow\frac{dV}{dt}=-0.15$ **or** compares $\frac{dV_{out}}{dt}$ $(=0.6)$ against $\frac{dV_{in}}{dt}$ $(=0.45)$ at $V=2$ (7) $V=2\Rightarrow "1.5"-"1.25"e^{-"0.3"t}=2\Rightarrow e^{-"0.3"t}<0$ (8) $"1.5"-"1.25"e^{-"0.3"t}=2\Rightarrow \ln(-0.4)$ is undefined (condone e.g. gives a maths error)	M1	3.2a
	• The (upper) limit for V is 1.5 (m^3) so no (the container will not become full) (first 4 bullets) • If $V = 2$ (or $V > 1.5$), it would be emptying so no (it can never be full) (bullets 5, 6) • No, as the equation cannot be solved (or is not true/doesn't work) when $V = 2$ (bullets 7, 8)	A1ft	2.4
		(2)	
			(9 marks)

Notes

(c)

M1: See main scheme. If using the answer to part (b) it must be of the form $V = P - Qe^{-kt}$ but there is no limitation on the values of their P, Q or k.
Substitution of a large value for t may score this mark but it is unlikely to be recovered to score the A1 unless they reference e.g. V_{max} being "1.5".
Reference to an (upper) limit of "1.5" or their P can imply the method mark.
If setting $V = 2$ in their equation they must reach either ln(–ve) **or** solve the equation to reach a value for t to score this mark.

A1ft: Must conclude "no" or equivalent e.g. "the container will not become full".
Makes a correct interpretation for their method (see bullets 1-8) with a clear conclusion e.g. "no".
To score this mark through ft, their V must be of the form $V = P - Qe^{-kt}$ with $k > 0$, $Q > 0$ **and** $0 < P < 2$ if used (but note that they can still use the answer to part (a) to score both marks via bullets 1, 3, 5 or 6). Allow "it" in place of the "container"/"tank".
Just stating "the equation cannot be solved when $V = 2$" without any evidence is M0A0.
There must be no incorrect working if solving their equation or contradictory statements such as "t cannot be negative" but condone notational errors provided the intention is clear.