Question	Scheme	Marks	AOs
13(i)	$(n =)\ 3$	B1	2.2a
		(1)	
(ii)	30 solutions **or** e.g. each interval of length π has 6 solutions	B1	2.2a
	e.g., Each interval of length π has 6 solutions, and there are 5 intervals of length π, so $5\times 6 = 30$ solutions.	dB1	2.4
		(2)	
			(3 marks)

Notes

(i)

B1: cao $(n =)\ 3$

(ii)

B1: Either:

- Deduces 30 solutions
- **or** deduces the correct **total** number of solutions for $\sin(nx) = k$ **and** $\sin(nx) = -k$ (or $\sin^2(nx) = k^2$) in any relevant interval $(0 \text{ to } \pi, 2\pi, 4\pi \text{ or } 5\pi)$
- **or** deduces the correct number of solutions in 0 to 5π for either $\sin(nx) = k$ or $\sin(nx) = -k$
- **or** deduces the correct **total** number of solutions for $\sin(x) = k$ **and** $\sin(x) = -k$ in any relevant interval $(0 \text{ to } \pi, 2\pi, 4\pi \text{ or } 5\pi)$

dB1: Requires 30 (solutions) and correct justification comprising all the elements in **one** of the bullet points below. See the tables on the next page for the correct values.

- Deduces the correct **total** number of solutions for $\sin(nx) = k$ **and** $\sin(nx) = -k$ (or $\sin^2(nx) = k^2$) in any relevant interval $(0 \text{ to } \pi, 2\pi \text{ or } 4\pi)$ **and** scales the interval to $0 \text{ to } 5\pi$
- **or** deduces the correct number of solutions in 0 to 5π for $\sin(nx) = k$ **and** for $\sin(nx) = -k$ and then adds
- **or** finds the **total** correct number of solutions for $\sin(x) = k$ **and** $\sin(x) = -k$ in any relevant interval $(0 \text{ to } \pi, 2\pi, 4\pi \text{ or } 5\pi)$ **and** then scales the interval to $0 \text{ to } 5\pi$ **and** multiplies by 3

Just stating e.g. $2\times 6\times 2.5$ is insufficient for the dB1 mark without further justification. There are many acceptable variations, and some examples are below and on the next page.

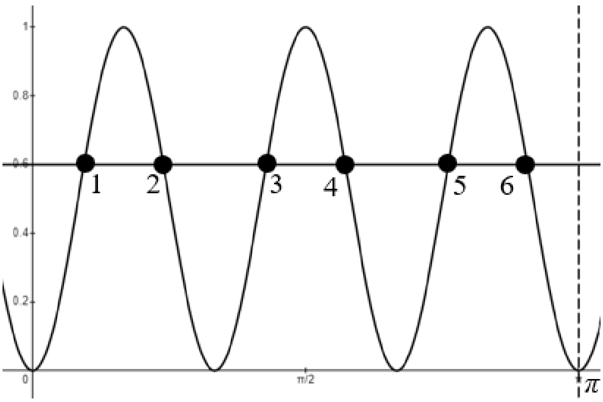
Some examples:

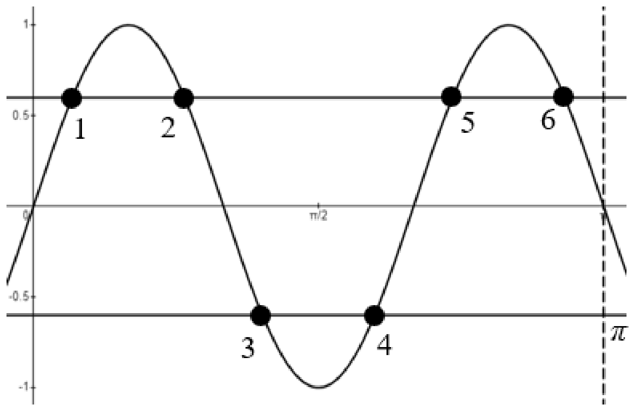
- 6 solutions in each interval of length π (B1) **and** 5 intervals of length π so 30 (dB1)
- 12 solutions in each interval of length 2π (B1) **and** 2.5 intervals of length 2π so 30 (dB1)
- 2 solutions for $\sin x = k$ up to 2π, so for $\sin^2 x$ there are 4 per 2π (B1) **and** since $0 \leqslant x < 5\pi$ then $0 \leqslant 3x < 15\pi$ so $\dfrac{15}{2}\times 4 = 30$ (dB1)
- Double the 6 solutions since $\sin(nx) = \pm k$ (or $\sin^2(nx) = k^2$) so 12 (B1) **and** so 24 in 4π, hence 30 in 5π (dB1)
- 16 solutions for $\sin(nx) = k$ (B1) **and** 14 (solutions) for $\sin(nx) = -k \rightarrow 30$ (dB1)
- $\sin^2(3x) = \dfrac{1-\cos(6x)}{2}$ and as $\cos x = k$ has 5 solutions in 0 to 5π (B1) **and** so $\cos 6x$ has $5\times 6 = 30$ (dB1)

Graphical approaches may be used but these must be convincing, clearly showing the correct number of solutions in one of the relevant intervals. Be lenient with the shape of the curve.

e.g. 30 solutions in $0 \leq x < 5\pi$ [with 6 solutions on, for example, either sketch below]

$y = \sin^2(3x)$ $0 \leq x < \pi$ $y = \sin(3x)$ $0 \leq x < \pi$





Note that a suitable graph of sin*x* showing, e.g. 10 solutions up to 5π, followed by 30 (calculation of ×3 clearly implied) would also be eligible to score the dB1.

The following are examples of an incorrect justification but scores B1dB0 for reaching 30:

- $\sin(nx) = k$ has 3×6 solutions in the interval 0 to 5π, $\sin(nx) = -k$ has 3×4 solutions (in the interval 0 to 5π) so 18 + 12 = 30 solutions in total.
- $\sin(nx) = k$ has 15 solutions in the interval 0 to 5π, so for $\sin^2(nx)$ has 15×2 = 30 solutions in total.

For reference, the tables below show the total number of solutions for each branch of $\sin^2(nx) = k^2$ with $n = 3$ and the bold values score the first B1 **provided they are in the correct interval**.

	$0 \leq x < \pi$	$0 \leq x < 2\pi$	$0 \leq x < 4\pi$	$0 \leq x < 5\pi$
$\sin(nx) = k$	4	6	12	**16**
$\sin(nx) = -k$	2	6	12	**14**
Total solutions	**6**	**12**	**24**	**30**

and $n = 1$ for those that deal with nx ($3x$) last.

	$0 \leq x < \pi$	$0 \leq x < 2\pi$	$0 \leq x < 4\pi$	$0 \leq x < 5\pi$
$\sin(x) = k$	2	2	4	6
$\sin(x) = -k$	0	2	4	4
Total solutions	**2**	**4**	**8**	**10**