Question	Scheme	Marks	AOs
14(a)	$\left(\overrightarrow{AD}=\overrightarrow{AB}-\overrightarrow{DB}=\right)\ 2\mathbf{a}+3\mathbf{b}-\left(-4\mathbf{a}+k\mathbf{b}\right)\ \ \left(=6\mathbf{a}+\left(3-k\right)\mathbf{b}\right)$	M1	1.1b
	$\frac{15}{6}=2.5\Rightarrow\frac{-5}{3-k}=2.5\rightarrow k=\ldots$	dM1	1.1b
	$k=5$ *	A1*	2.1
		(3)	

Notes

(a) **Note: Condone the use of column vectors throughout this question.**
There may be working on the diagram that can be awarded marks.

M1: Attempts either $\left(\overrightarrow{AD}=\overrightarrow{AB}-\overrightarrow{DB}=\right)\ 2\mathbf{a}+3\mathbf{b}-\left(-4\mathbf{a}+k\mathbf{b}\right)$ or $\left(\overrightarrow{DA}=\right)\ -4\mathbf{a}+k\mathbf{b}-\left(2\mathbf{a}+3\mathbf{b}\right)$ or $\left(\overrightarrow{DB}=\overrightarrow{DA}+\overrightarrow{AB}=\right)\ \alpha\left(15\mathbf{a}-5\mathbf{b}\right)+2\mathbf{a}+3\mathbf{b}$

Allow subtraction either way round and may be implied by one correct component or by e.g. $2\mathbf{a}+3\mathbf{b}=\overrightarrow{AD}-4\mathbf{a}+k\mathbf{b}$

For reference: $\overrightarrow{AD}=6\mathbf{a}+\left(3-k\right)\mathbf{b}$ and $\overrightarrow{DA}=-6\mathbf{a}+\left(k-3\right)\mathbf{b}$

Allow e.g. $\left(\overrightarrow{AD}=\right)\begin{pmatrix}6\mathbf{a}\\ \left(3-k\right)\mathbf{b}\end{pmatrix}$ or $\left(\overrightarrow{AD}=\right)\begin{pmatrix}6\\ 3-k\end{pmatrix}$ including without the brackets $\begin{matrix}6\\ 3-k\end{matrix}$

Condone the use of gradients or ratios for $\overrightarrow{AD}$ e.g. $\frac{6}{3-k}$ or "6 : 3 – k" to imply this mark (either way round).

There are alternatives using e.g. $\overrightarrow{DC}$ but, in these cases, we require two expressions for the same vector, of which one expression must use $\overrightarrow{DB}$.

dM1: A full method to solve the problem. Some possible approaches:

- attempts to find a scale factor and uses it to find k
- sets up equivalent fractions e.g. $\frac{15}{6}=-\frac{5}{3-k}$ or e.g. $\frac{2--4}{3-k}=\frac{15}{-5}$ and solves for k
- sets up equivalent ratios e.g. 6 : 15 = 3 – k : –5 and solves for k
- sets up simultaneous equations and solves for k e.g. $2\mathbf{a}+3\mathbf{b}+4\mathbf{a}-k\mathbf{b}=\alpha\left(15\mathbf{a}-5\mathbf{b}\right)$ o.e. **or** $\left(\overrightarrow{DB}=\overrightarrow{DA}+\overrightarrow{AB}=\right)\ \alpha\left(-15\mathbf{a}+5\mathbf{b}\right)+2\mathbf{a}+3\mathbf{b}=-4\mathbf{a}+k\mathbf{b}$ leading to $6=15\alpha$ $\left(\alpha=\frac{2}{5}\right)$ and $3-k=-5\alpha$ hence $k=\ldots$

Note that their coefficient might be the reciprocal, from e.g. $\beta\left(2\mathbf{a}+3\mathbf{b}+4\mathbf{a}-k\mathbf{b}\right)=15\mathbf{a}-5\mathbf{b}$ $\left(\beta=2.5\right)$ and directions might be reversed in which case $\alpha=-0.4$ or $\beta=-2.5$ can be used.

A1*: Arrives at $k=5$ via a correct method. Usually this will be following:

- A correct expression for $\overrightarrow{AD}$ (e.g. $2\mathbf{a}+3\mathbf{b}-\left(-4\mathbf{a}+k\mathbf{b}\right)$) or $\overrightarrow{DA}$ or $\overrightarrow{BD}$ or $\overrightarrow{DB}$ which may be mislabelled.
- Correct scale factor stated (±2.5 or ±0.4) or implied (e.g., by $\frac{-5}{3-k}=\frac{15}{6}$ or $3\left(3-k\right)=-6$)
- A correct intermediate equation that leads to $k=5$

An example minimal response might look like:

e.g. $\alpha\left(15\mathbf{a}-5\mathbf{b}\right)=6\mathbf{a}+\left(3-k\right)\mathbf{b}\rightarrow6=15\alpha\rightarrow\alpha=\frac{2}{5}\rightarrow-5\left(\frac{2}{5}\right)=3-k\rightarrow k=5$

or e.g. $6\mathbf{a}+\left(3-k\right)\mathbf{b}\rightarrow\frac{15}{6}=-\frac{5}{3-k}\rightarrow k=5$

Condone missing/invisible brackets if recovered.

Alternative by verification:

M1: Sets $k = 5$, substitutes into $\overrightarrow{DB}$ and attempts $\left(\overrightarrow{AD} = \overrightarrow{AB} - \overrightarrow{DB} =\right)$ $2\mathbf{a} + 3\mathbf{b} - (-4\mathbf{a} + 5\mathbf{b})$ o.e.

dM1: Attempts to compare $\overrightarrow{AD}$ o.e. with $\overrightarrow{BC}$ (usually $6\mathbf{a} - 2\mathbf{b} = \alpha(15\mathbf{a} - 5\mathbf{b})$) and finds α

A1*: Requires:

- Correct $\overrightarrow{AD}$ o.e. e.g. $\overrightarrow{DA}$
- Correct scale factor $\alpha = \pm\frac{2}{5}$ or $\beta = \pm 2.5$ (sign dependent on their approach)
- Conclusion referencing the lines being parallel e.g. "hence $\overrightarrow{AD}$ and $\overrightarrow{BC}$ are parallel."

Question	Scheme	Marks	AOs
14(b)	e.g., $\left(\overrightarrow{BN}=\right)\ \frac{1}{5}\overrightarrow{BC}\ \left(=3\mathbf{a}-\mathbf{b}\right)$	B1	2.2a
	e.g., $\left(\overrightarrow{BX}=\lambda\overrightarrow{BD}=\right)\ \lambda\left(4\mathbf{a}-5\mathbf{b}\right)$	M1	2.1
	e.g., $\left(\overrightarrow{BX}=\lambda\overrightarrow{BD}=\right)\ \lambda\left(4\mathbf{a}-5\mathbf{b}\right)$ **and** e.g. $\left(\overrightarrow{BX}=\overrightarrow{BA}+\mu\overrightarrow{AN}=\right)\ \left(-2\mathbf{a}-3\mathbf{b}\right)+\mu\left(2\mathbf{a}+3\mathbf{b}+"3\mathbf{a}-\mathbf{b}"\right)$	dM1	3.1a
	$4\lambda=-2+5\mu$, $-5\lambda=-3+2\mu$ $\Rightarrow \lambda=\ldots\left(\frac{1}{3}\right)$ or $\mu=\ldots\left(\frac{2}{3}\right)$	ddM1	1.1b
	$1:2$	A1	2.2a
		(5)	
		(8 marks)	

Notes

(b) **Note: Condone the use of column vectors throughout this question.**
There may be working on the diagram that can be awarded marks.

B1: Deduces a correct interpretation of the ratio $BN:NC=1:4$ that enables a start to a solution.
i.e., progresses to a correct statement that is at least as far as $\left(\overrightarrow{BN}=\right)\ \frac{1}{5}\overrightarrow{BC}$ (or e.g. $3\mathbf{a}-\mathbf{b}$)
or $\left(\overrightarrow{CN}=\right)\ \frac{4}{5}\overrightarrow{CB}$ (or e.g. $4\mathbf{b}-12\mathbf{a}$). May be embedded in e.g. $\left(\overrightarrow{AN}=\right)\ 2\mathbf{a}+3\mathbf{b}+\frac{1}{5}\overrightarrow{BC}$
Allow e.g. $\frac{1}{5}\begin{pmatrix}15\\-5\end{pmatrix}$ or $\begin{pmatrix}3\\-1\end{pmatrix}$ or $\begin{pmatrix}3a\\-b\end{pmatrix}$ for this mark.

M1: For the key step in attempting a valid expression for $\overrightarrow{BX}$ (**or** $\overrightarrow{AX}$ **or** $\overrightarrow{DX}$ **or** $\overrightarrow{CX}$ **or** $\overrightarrow{NX}$) in terms of **a** and **b**. See diagram/notes on the next page for helpful vectors.
Condone slips provided their intention is clear.
Note that they might be using λ and μ the other way round or alternative variables.

Note: May be seen as a single expression such as $\overrightarrow{AB}=\overrightarrow{AX}+\overrightarrow{XB}$ $\left(\text{i.e. } \overrightarrow{AB}=p\overrightarrow{AN}+q\overrightarrow{DB}\right)$
or $\overrightarrow{BN}=\overrightarrow{BX}+\overrightarrow{XN}$ $\left(\text{i.e. } \overrightarrow{BN}=p\overrightarrow{BD}+q\overrightarrow{AN}\right)$ either of which scores M1dM1 simultaneously.

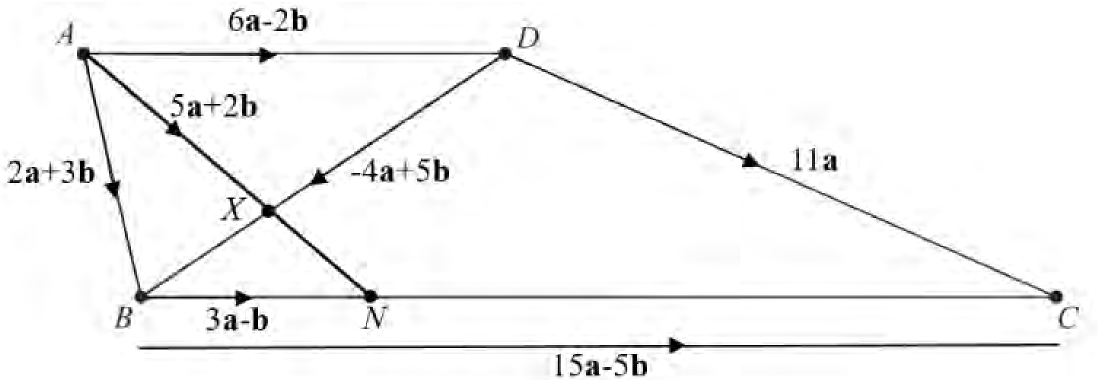
dM1: For the key step in attempting a **second** valid expression for their $\overrightarrow{BX}$ (**or** $\overrightarrow{AX}$ **or** $\overrightarrow{DX}$ **or** $\overrightarrow{CX}$ **or** $\overrightarrow{NX}$) in terms of **a** and **b** which enables the problem to be solved, i.e., it must not be parallel in approach to the first. See diagram/notes on the next page for helpful vectors.
One expression should involve $"\lambda"\left(4\mathbf{a}-5\mathbf{b}\right)$ and the other should involve $"\mu"\left("5\mathbf{a}+2\mathbf{b}"\right)$
Condone slips provided their intention is clear.
They must use different parameters in their approaches, e.g., λ and μ.
If using e.g. $\overrightarrow{DX}=-6\mathbf{a}+2\mathbf{b}+\mu\left(5\mathbf{a}+2\mathbf{b}\right)$ and $\overrightarrow{XB}=\lambda\left(-4\mathbf{a}+5\mathbf{b}\right)$ this mark is not scored until they set $\overrightarrow{DX}+\overrightarrow{XB}=\overrightarrow{DB}$ as $-6\mathbf{a}+2\mathbf{b}+\mu\left(5\mathbf{a}+2\mathbf{b}\right)+\lambda\left(-4\mathbf{a}+5\mathbf{b}\right)=-4\mathbf{a}+5\mathbf{b}$
Dependent on the previous method mark.

ddM1: Compares coefficients of **a** and **b** to create simultaneous equations in their parameters and attempts to solve (which may be by calculator) leading to a value for one of their parameters.
Condone slips provided the intention is clear.
This mark may be implied by a **correct** value for e.g. λ following their two **correct** expressions for e.g. $\overrightarrow{BX}$
Dependent on both previous method marks.

A1: $1:2$ o.e. Must follow a correct value for their parameter.
The correct ratio seen does **not** imply full marks. Candidates must show detailed reasoning.

Allow equivalent ratios e.g. $\frac{1}{3}:\frac{2}{3}$ and ISW (e.g. $1:3$) but they must be the correct way round.
There may be attempts using similar triangles. Send to review.

Helpful Diagram:



Note: Some examples of valid expressions for the M and dM marks for part (b) are:
In each expression they may use different parameters and e.g. $1-\lambda$ might just be e.g. ϕ.

- $\overrightarrow{BX} = \lambda\overrightarrow{BD} = \lambda(4\mathbf{a}-5\mathbf{b})$
- $\overrightarrow{BX} = \overrightarrow{BA} + \mu\overrightarrow{AN} = (-2\mathbf{a}-3\mathbf{b}) + \mu(2\mathbf{a}+3\mathbf{b}+\text{"}3\mathbf{a}-\mathbf{b}\text{"})$
- $\overrightarrow{BX} = \overrightarrow{BN} + (1-\mu)\overrightarrow{NA} = (\text{"}3\mathbf{a}-\mathbf{b}\text{"}) + (1-\mu)(-2\mathbf{a}-3\mathbf{b}+\text{"}-3\mathbf{a}+\mathbf{b}\text{"})$
- $\overrightarrow{DX} = (1-\lambda)\overrightarrow{DB} = (1-\lambda)(-4\mathbf{a}+5\mathbf{b})$
- $\overrightarrow{DX} = \overrightarrow{DA} + \mu\overrightarrow{AN} = (\text{"}-6\mathbf{a}+2\mathbf{b}\text{"}) + \mu(2\mathbf{a}+3\mathbf{b}+\text{"}3\mathbf{a}-\mathbf{b}\text{"})$
- $\overrightarrow{DX} = \overrightarrow{DN} + (1-\mu)\overrightarrow{NA} = (\text{"}-\mathbf{a}+4\mathbf{b}\text{"}) + (1-\mu)(-2\mathbf{a}-3\mathbf{b}+\text{"}-3\mathbf{a}+\mathbf{b}\text{"})$
- $\overrightarrow{AX} = \mu\overrightarrow{AN} = \mu(2\mathbf{a}+3\mathbf{b}+\text{"}3\mathbf{a}-\mathbf{b}\text{"})$
- $\overrightarrow{AX} = \overrightarrow{AB} + \lambda\overrightarrow{BD} = (2\mathbf{a}+3\mathbf{b}) + \lambda(4\mathbf{a}-5\mathbf{b})$
- $\overrightarrow{AX} = \overrightarrow{AD} + (1-\lambda)\overrightarrow{DB} = (\text{"}6\mathbf{a}-2\mathbf{b}\text{"}) + (1-\lambda)(-4\mathbf{a}+5\mathbf{b})$
- $\overrightarrow{XN} = \mu\overrightarrow{AN} = \mu(2\mathbf{a}+3\mathbf{b}+\text{"}3\mathbf{a}-\mathbf{b}\text{"})$
- $\overrightarrow{XN} = \lambda\overrightarrow{DB} + \overrightarrow{BN} = \lambda(-4\mathbf{a}+5\mathbf{b}) + (\text{"}3\mathbf{a}-\mathbf{b}\text{"})$
- $\overrightarrow{XN} = (1-\lambda)\overrightarrow{BD} + \overrightarrow{DN} = (1-\lambda)(4\mathbf{a}-5\mathbf{b}) + (\text{"}-\mathbf{a}+4\mathbf{b}\text{"})$

or alternatives using C or N as starting points, but these are unlikely.