10. Water flows at a constant rate into a large container.

There is a tap at the bottom of the container.

At time t hours after the tap was opened

- the volume of water in the container is $V\,\text{m}^3$
- water is flowing into the container at a constant rate of $0.45\,\text{m}^3$ per hour
- water is leaving the container through the tap at a rate of $0.3V\,\text{m}^3$ per hour

(a) Show that

$$20\frac{\mathrm{d}V}{\mathrm{d}t} = 9 - 6V$$

(2)

Given that when the tap was opened, there was $0.25\,\text{m}^3$ of water in the container,

(b) solve the differential equation to show that

$$V = P - Q\mathrm{e}^{-kt}$$

where P, Q and k are positive constants to be found.

(5)

Given that

- the capacity of the container is $2\,\text{m}^3$
- the tap remains open
- the water continues to flow into the tank at the same rate

(c) determine whether the container will ever become full, giving a reason for your answer.

(2)