15.

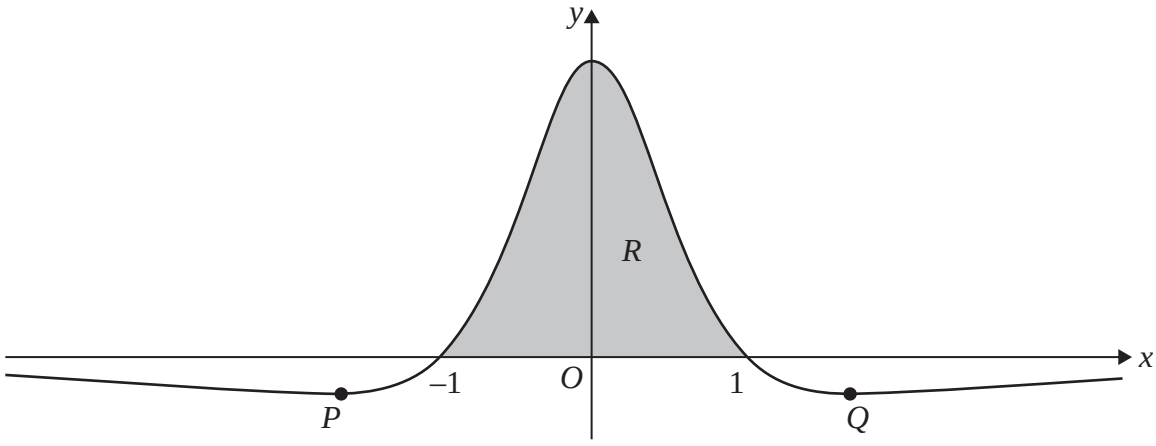

Figure 5

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

Figure 5 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \frac{1 - x^2}{\left(1 + x^2\right)^2}$$

The curve

- intersects the x-axis at -1 and 1
- has minimum turning points at P and Q

as shown in Figure 5.

(a) Use calculus to find the exact coordinates of P.

(5)

(b) Using the substitution $x = \tan\theta$ show that

$$\int_{-1}^{1} \mathrm{f}(x)\,\mathrm{d}x = \int_{\alpha}^{\beta} \cos 2\theta\,\mathrm{d}\theta$$

where α and β are constants to be found.

(5)

The finite region R, shown shaded in Figure 5, is bounded by the x-axis and the curve.

(c) Use algebraic integration to find the area of R.

(3)