

$$\int \ln\left(\frac{x^2+1}{x^2}\right) \times \frac{\sqrt{x^2+1}}{x^4} dx$$

M1

$$\int \frac{\sqrt{x^2+1}}{\sqrt{x^2}} \ln\left(1 + \frac{1}{x^2}\right) \times \frac{1}{x^3} dx$$

M1

$$\int \sqrt{1 + \frac{1}{x^2}} \ln\left(1 + \frac{1}{x^2}\right) \times \frac{1}{x^3} dx$$

$$u = \sqrt{1 + \frac{1}{x^2}}$$

M1

$$u^2 = 1 + \frac{1}{x^2}$$

M1

$$2u du = -\frac{2}{x^3} dx$$

$$\int u \ln(u^2) \times \frac{1}{x^3} \times (-x^3) du$$

M1

$$\int -u^2 \ln u^2 du$$

M1

$$-2 \int u^2 \ln u du$$

Attempts to use integration by parts,

M1

$$t = \ln u \Rightarrow \frac{dt}{du} = \frac{1}{u}$$

$$\frac{dv}{du} = u^2 \Rightarrow v = \frac{u^3}{3}$$

$$-2 \left(\frac{u^3}{3} \ln u - \int \frac{u^3}{3} \times \frac{1}{u} du \right)$$

M1

$$-\frac{2u^3}{3} \ln u + \frac{2u^3}{9} + C$$

M1

$$\frac{2}{9} u^3 (1 - 3 \ln u) + C$$

M1

$$\frac{2}{9} \left(\sqrt{1 + \frac{1}{x^2}} \right)^3 \left(1 - 3 \ln \left(\sqrt{1 + \frac{1}{x^2}} \right) \right) + C$$

M1

$$\frac{2}{9x^3} (x^2 + 1)^{\frac{3}{2}} \left(1 - 3 \ln \left(\sqrt{\frac{x^2 + 1}{x^2}} \right) \right) + C$$

B1

[12]

[12 marks]