

4(a)

Write as $y^2 = \frac{x^2}{x - \frac{1}{2}}$ and since $x > 0$,

$$y = \frac{x}{\left(x - \frac{1}{2}\right)^{\frac{1}{2}}}$$

$$\frac{dy}{dx} = \frac{\left(x - \frac{1}{2}\right)^{\frac{1}{2}} \times 1 - x \times \frac{1}{2} \left(x - \frac{1}{2}\right)^{-\frac{1}{2}}}{\left(\left(x - \frac{1}{2}\right)^{\frac{1}{2}}\right)^2}$$

$$\frac{dy}{dx} = \frac{(x - 1)}{2 \left(x - \frac{1}{2}\right)^{\frac{3}{2}}}$$

Let P be a generic point on the curve for which $y > 0$

Then,

$$\left. \frac{dy}{dx} \right|_{x=P} = \frac{P - 1}{2 \left(P - \frac{1}{2}\right)^{\frac{3}{2}}}$$

Equation of tangent at P takes the form,

$$y - \frac{P}{\left(P - \frac{1}{2}\right)^{\frac{1}{2}}} = \frac{P - 1}{2 \left(P - \frac{1}{2}\right)^{\frac{3}{2}}} (x - P)$$

Substitute $x = \frac{1}{2}$ and $y = \sqrt{2}$

$$\sqrt{2} - \frac{P}{\left(P - \frac{1}{2}\right)^{\frac{1}{2}}} = \frac{P - 1}{2 \left(P - \frac{1}{2}\right)^{\frac{3}{2}}} \left(\frac{1}{2} - P\right)$$

$$\sqrt{2} - \frac{P}{\left(P - \frac{1}{2}\right)^{\frac{1}{2}}} = \frac{1 - P}{2 \left(P - \frac{1}{2}\right)^{\frac{3}{2}}}$$

$$2\sqrt{2} \left(P - \frac{1}{2}\right)^{\frac{1}{2}} - 2P = 1 - P$$

$$8 \left(P - \frac{1}{2}\right) = P^2 + 2P + 1$$

$$P^2 - 6P + 5 = 0$$

$$(P - 1)(P - 5) = 0$$

$$P = 1 \text{ or } 5$$

Hence, two tangents in the form

$$y - \frac{P}{\left(P - \frac{1}{2}\right)^{\frac{1}{2}}} = \frac{P - 1}{2 \left(P - \frac{1}{2}\right)^{\frac{3}{2}}} (x - P)$$

where $P = 1$ or 5

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[11]

4(b)Substitutes $P = 1$,

$$y - \frac{1}{\left(1 - \frac{1}{2}\right)^{\frac{1}{2}}} = \frac{1 - 1}{2\left(1 - \frac{1}{2}\right)^{\frac{3}{2}}}(x - 1)$$

$$y - \sqrt{2} = 0$$

Substitutes $P = 5$,

$$y - \frac{5}{\left(5 - \frac{1}{2}\right)^{\frac{1}{2}}} = \frac{5 - 1}{2\left(5 - \frac{1}{2}\right)^{\frac{3}{2}}}(x - 5)$$

$$y - \frac{5\sqrt{2}}{3} = \frac{4}{\left(\frac{27}{\sqrt{2}}\right)}(x - 5)$$

$$4\sqrt{2}x - 27y + 25\sqrt{2} = 0 \quad \text{or} \quad 8x - 27\sqrt{2}y + 50 = 0$$

M1

A1

M1

M1

A1

[5]

[16 marks]