

$$Q = \frac{1}{2}Y = (-14, 2, -6)$$

M1

$$R = \frac{1}{4}X = (-6, 18, 6)$$

M1

$$\overrightarrow{RP} = k\overrightarrow{RY} = k(\mathbf{y} - \mathbf{r})$$

M1

$$\overrightarrow{RP} = k((-28, 4, -12) - (-6, 18, 6)) = (-22k, -14k, -18k)$$

M1

$$\begin{aligned}\overrightarrow{QP} &= \overrightarrow{QO} + \overrightarrow{OR} + \overrightarrow{RP} \\ &= -(-14, 2, -6) + (-6, 18, 6) + (-22k, -14k, -18k) \\ &= (-22k + 8, -14k + 16, -18k + 12)\end{aligned}$$

M1

$$\begin{aligned}\overrightarrow{PX} &= \overrightarrow{PR} + \overrightarrow{RX} \\ &= -(-22k, -14k, -18k) + 3(-6, 18, 6) \\ &= (22k - 18, 14k + 54, 18k + 18)\end{aligned}$$

M1

$$\begin{aligned}P, Q \text{ and } X \text{ are colinear, so } \overrightarrow{QP} &= \lambda\overrightarrow{PX} \\ (-22k + 8, -14k + 16, -18k + 12) &= \lambda(22k - 18, 14k + 54, 18k + 18)\end{aligned}$$

M1

Equate any two components,
e.g. $-22k + 8 = 22\lambda k - 18\lambda$
and $-14k + 16 = 14\lambda k + 54\lambda$

M1

Multiply the second equation by $\frac{11}{7}$, and then equate

M1

$$-18\lambda - 8 = \frac{594}{7}\lambda - \frac{176}{7}$$

$$\lambda = \frac{1}{6}$$

6 cont.

Substitute to find k ,

$$\text{e.g. } -22k - 22\left(\frac{1}{6}\right)k = -18\left(\frac{1}{6}\right) - 8$$

$$k = \frac{3}{7}$$

$$\overrightarrow{OP} = \overrightarrow{OR} + \overrightarrow{RP}$$

$$= (-6, 18, 6) + \left(-22\left(\frac{3}{7}\right), -14\left(\frac{3}{7}\right), -18\left(\frac{3}{7}\right)\right)$$

$$= \left(-\frac{108}{7}, 12, -\frac{12}{7}\right) = P$$

M1

M1

[11]

[11 marks]