Question	Scheme	Marks	AOs
3(a)	$\frac{1}{2}(65)(2a+64d)=4940$	M1	1.1b
	$\Rightarrow 65a+2080d=4940$ $\Rightarrow a+32d=76$ *	A1*	2.1
		(2)	
(b)	$a+48d=64$	B1	1.1b
		(1)	
(c)	$\begin{array}{c} a+32d=76 \\ "a+48d=64" \end{array} \Rightarrow a=100,\ d=-\frac{3}{4}$	M1 A1	1.1b 1.1b
		(2)	

Mark (a), (b), (c) and (d) together.

(a)

M1: Uses a correct sum formula in terms of a and d with n = 65 and sets = 4940

Must be in terms of a and d and not e.g. $\frac{1}{2}(65)(a+l)=4940$

Condone invisible brackets for this mark as long as the correct equation is implied by subsequent work which is not the given answer e.g.

$\frac{1}{2}(65)\times 2a+64d=4940 \Rightarrow 65a+2080d=4940$

Condone a missing trailing bracket e.g. $\frac{1}{2}(65)(2a+64d=4940$

A1*: Achieves the printed answer with no errors.

An intermediate line is **not** required so it is acceptable to go from $\frac{1}{2}(65)(2a+64d)=4940$ to $a+32d=76$

Do **not** condone **invisible brackets** e.g. $\frac{1}{2}(65)\times 2a+64d=4940$ even if recovered but condone a missing **trailing bracket** e.g. $\frac{1}{2}(65)(2a+64d=4940$

Allow e.g. $32d+a=76$ or $76=a+32d$ or $76=32d+a$

(b)

B1: Any correct equation e.g. $a+(49-1)d=64$

(c)

M1: Solves $a+32d=76$ simultaneously with their equation in a and d only from part (b) to obtain values for a and d. Condone miscopies of the equations as long as the intention is clear.
You do not need to check their working or values as long as they reach values for a and d which may just be written down e.g. using a calculator – you do not need to check.

A1: Correct values and isw once correct values are seen.

Allow equivalents for $-\frac{3}{4}$ e.g. -0.75

Special case: If you see both correct answers the M1A1 can be implied. This does not depend on the B mark in part (b).

(d) Note that the question says "Solutions relying entirely on calculator technology are not acceptable" so the correct answer with no working in (d) scores no marks.

Note that to score marks in (d) their *d* must be <u>negative</u> otherwise there is no maximum.

Note that there are many different ways to answer (d). Apply the method that best fits their approach.

In all methods, apart from Trial and Improvement,

- **M1 is for a valid method to identify the value of *n* that gives the maximum sum**
- **dM1 for attempting the sum with <u>a positive integer value</u>** within 2 either below or above their value of *n*
- **A1 for the correct value**

Main Scheme

3(d)	$a+(n-1)d ... 0 \Rightarrow 100-\frac{3}{4}(n-1) ... 0 \Rightarrow n....(134.333)$	M1	3.1a
	$S_{134}=\frac{134}{2}\left(200-\frac{3}{4}\times 133\right)= ...$ oe e.g. $\frac{134}{2}\left(100+100-\frac{3}{4}\times 133\right)= ...$	**d**M1	1.1b
	$=6716.75$	A1	1.1b
		(3)	

Notes

M1: Realises that the maximum occurs immediately before the terms become negative and so attempts to find the value of *n* when this happens.
It is for an attempt to solve $a+(n-1)d ... 0$ or $a+nd ... 0$ for *n* with their values of *a* and d where $d<0$ and "…" is "=" or any inequality.
Note: e.g. $a+(n-1)d=0 \Rightarrow 100-\frac{3}{4}n-\frac{3}{4} \Rightarrow n= ...$ scores M1 but $100-\frac{3}{4}n-\frac{3}{4} \Rightarrow n= ...$ with no correct term formula quoted scores M0
May be implied by sight of 134.333… **but not just 134 without working seen.**

dM1: Uses a correct sum formula with their *a*, *d* and a **positive integer** within 2 either below or above their value of *n* to find the greatest sum.
E.g. if they get $n = 134.333...$ they can use 133, 134, 135 or 136
May be implied by a correct value for the sum.
Depends on the first method mark.

A1: Correct value from fully correct work condoning slips in notation.
Allow equivalents e.g. $\frac{26867}{4}$ and isw once the correct answer is seen.
If more than one answer is given they must select the correct one.

Alternatives For M1:

Calculus:			
	$S_n=\frac{n}{2}\left(200-\frac{3}{4}\times(n-1)\right)=100.375n-0.375n^2$ $\frac{\mathrm{d}S_n}{\mathrm{d}n}=100.375-0.75n=0\Rightarrow n=133.83...$	M1	3.1a
Forms an expression using a correct sum formula in terms of *n* using their *a* and *d* and then differentiates their S_n with respect to *n* to obtain a linear expression in *n*, sets = 0 and solves for *n*. Cannot be implied by sight of just *n* = 133.83…			
	Completing the square		
	$S_n=\frac{n}{2}\left(200-\frac{3}{4}\times(n-1)\right)=\frac{803}{8}n-\frac{3}{8}n^2=-\frac{3}{8}\left(n^2-\frac{803}{3}n\right)$ $=-\frac{3}{8}\left(\left(n-\frac{803}{6}\right)^2-\frac{644809}{36}\right)$ So max when $n=\frac{803}{6}(=133.83...)$	M1	3.1a
Forms an expression using a correct sum formula in terms of *n* using their *a* and *d* and then attempts to complete the square to find a value for *n*. It requires at least $\alpha n^2+\beta n\to\alpha\left(n^2+kn\right)\to\alpha\left(\left(n+\frac{k}{2}\right)^2...\right)\to n=-\frac{k}{2}$ Cannot be implied by sight of just *n* = 133.83…			
Using the quadratic in *n*.			
	$S_n=\frac{n}{2}\left(200-\frac{3}{4}\times(n-1)\right)=\frac{803}{8}n-\frac{3}{8}n^2$ Max occurs when $n="-\frac{b}{2a}"=-\frac{803}{8}\div\left(-\frac{6}{8}\right)=\frac{803}{6}$	M1	3.1a
Forms an expression using a correct sum formula in terms of *n* using their *a* and *d* and then attempts to find $n="-\frac{b}{2a}"$ or equivalent i.e. the midpoint of the roots. It requires at least $\alpha n^2+\beta n\to n=-\frac{\beta}{2\alpha}$ Cannot be implied by sight of just *n* = 133.83… **Note that if they work with the quadratic in *n* as above, there are no marks for <u>just</u> solving it = 0, e.g. they must consider $"-\frac{b}{2a}"$ or complete the square or use calculus etc.**			

The subsequent dM1A1 can then be applied according to the main scheme.

(d) Alternative using trial and improvement with correct *a* and *d*:			
	$S_{133}=\frac{133}{2}\left(200-\frac{3}{4}\times(133-1)\right)=...(6716.5)$ **or** $S_{134}=\frac{134}{2}\left(200-\frac{3}{4}\times(134-1)\right)=...(6716.75)$ **or** $S_{135}=\frac{135}{2}\left(200-\frac{3}{4}\times(135-1)\right)=...(6716.25)$	M1	3.1a
	$S_{133}=\frac{133}{2}\left(200-\frac{3}{4}\times(133-1)\right)=...(6716.5)$ **and** $S_{134}=\frac{134}{2}\left(200-\frac{3}{4}\times(134-1)\right)=...(6716.75)$ **and** $S_{135}=\frac{135}{2}\left(200-\frac{3}{4}\times(135-1)\right)=...(6716.25)$	dM1	1.1b
	So maximum is 6716.75	A1	1.1b

M1: Uses a correct sum formula to find the sum when n = 133 **or** 134 **or** 135
May be implied by a correct value for S_{133} or S_{134} or S_{135}

dM1: Uses a correct sum formula to find the sum when n = 133 **and** 134 **and** 135
May be implied by a correct value for S_{133} and S_{134} and S_{135}

A1: Correct value. Allow equivalents e.g. $\frac{26867}{4}$ and isw once the correct answer is seen.
It must be clear which of their 3 values is their chosen answer.

(8 marks)

Note that there may be other valid methods e.g. using trial and improvement via the terms:

$$u_{134}=100-\frac{3}{4}\times 133=0.25$$

$$u_{135}=100-\frac{3}{4}\times 134=-0.5$$

So largest sum occurs when *n* = 134

$$S_{134}=\frac{1}{2}(134)\left(2\times 100-\frac{3}{4}\times 133\right)=6716.75$$

Do not allow approaches that <u>just</u> find the maximum value of the sum

e.g. $S_n=\frac{n}{2}\left(200-\frac{3}{4}\times(n-1)\right)=100.375n-0.375n^2 \Rightarrow \text{Max}=\frac{644809}{96}\left(=6716.760417\right)$

scores M0dM0A0