Question	Scheme	Marks	AOs
4(a)	$u_1 = k \Rightarrow (u_2 =)\ 3k^2 - 1$	B1	1.1b
	$\Rightarrow (u_3 =)\ 3("3k^2 - 1")^2 - 1$	M1	1.1b
	$= 3(9k^4 - 6k^2 + 1) - 1 = 27k^4 - 18k^2 + 2$ *	A1*	2.1
		(3)	

Notes

Condone the omission of e.g. "$u_2 =$" and/or "$u_3 =$" and condone incorrect labelling as long as the intention is clear e.g. $u_1 = 3k^2 - 1,\ \ u_2 = 3("3k^2 - 1")^2 - 1$

B1: Deduces the correct second term $3k^2 - 1$ oe e.g. $3(k)^2 - 1$

May be implied by their attempt at u_3 e.g. $(u_3 =)\ 3(3k^2 - 1)^2 - 1$

M1: Uses the recurrence relation correctly with their u_2 to find u_3 in terms of k.

A1*: Obtains the printed answer with no errors and at least one correct intermediate line that is not the printed answer. Allow the terms to be written in any order.
The "$u_3 =$" is **not** required.

The $(3k^2 - 1)^2$ must be seen correctly expanded at some point but may be unsimplified e.g. $9k^4 - 3k^2 - 3k^2 + 1$ and may be seen embedded e.g. $3(3k^2 - 1)^2 - 1 = 27k^4 - 18k^2 + 3 - 1$

Do **not** condone invisible brackets e.g.
$(u_3 =) 3(3k^2 - 1)^2 - 1 = 3 \times 9k^4 - 6k^2 + 1 - 1 = 27k^4 - 18k^2 + 2$

Do **not** condone insufficient working e.g. $(u_3 =) 3(3k^2 - 1)^2 - 1 = 27k^4 - 18k^2 + 2$

But allow e.g. $u_1 = k \Rightarrow (u_2 =) 3k^2 - 1 \Rightarrow (u_3 =) 3(9k^4 - 6k^2 + 1) - 1 = 27k^4 - 18k^2 + 2$ as an intermediate line and $(3k^2 - 1)^2$ correctly expanded are both seen.

Note that some candidates may work in terms of "u" and then substitute for k at the end e.g. $u_2 = 3u_1{}^2 - 1 \Rightarrow u_3 = 3(3u_1{}^2 - 1)^2 - 1 = 3(9u_1{}^4 - 6u_1{}^2 + 1) - 1 = 27u_1{}^4 - 18u_1{}^2 + 2$
$u_1 = k \Rightarrow u_3 = 27k^4 - 18k^2 + 2$

In such cases, the B1 can be implied by $(u_2 =) 3u_1{}^2 - 1$ and the M1 by $(u_3 =) 3(3u_1{}^2 - 1)^2 - 1$ and then A1 for the correct expression with $u_1 = k$ substituted following sight of the correct expansion of $(3u_1{}^2 - 1)^2$

Question	Scheme	Marks	AOs
4(b)	$u_3 - 5u_2 = 11 \Rightarrow 27k^4 - 18k^2 + 2 - 5("3k^2 - 1") = 11$ $\Rightarrow 27k^4 - 33k^2 - 4 = 0$	M1	1.1b
	$27k^4 - 33k^2 - 4 = 0 \Rightarrow (9k^2 + 1)(3k^2 - 4) = 0 \Rightarrow k^2 = ...$	dM1	1.1b
	$k^2 = \dfrac{4}{3} \Rightarrow (k =) \pm \dfrac{2}{\sqrt{3}}$ oe	A1	2.2a
		(3)	
			(6 marks)

M1: Substitutes their u_2 and the given u_3 (or their u_3 provided it is of the same form i.e. $\alpha k^4+\beta k^2+\gamma,\ \alpha,\beta,\gamma\neq 0$) into $u_3-5u_2=11$ and rearranges to obtain a 3TQ in k^2. Condone miscopies of the expressions and slips with the rearranging as long as the intention is clear but do not condone miscopies of $u_3-5u_2=11$. The terms do not have to be all on one side but must be collected to 3 terms e.g. $27k^4-33k^2=4$. If they do gather terms to one side, the "= 0" may be implied by later work e.g. when they attempt to solve.

dM1: Depends on the first method mark.

Solves their 3TQ by any acceptable correct **non-calculator** method to obtain a **positive** value for k^2 which may be implied by their value for k e.g.

$$27k^4-33k^2-4=0\Rightarrow\left(9k^2+1\right)\left(3k^2-4\right)=0\Rightarrow k=(\pm)\frac{2\sqrt{3}}{3}$$

See General Guidance for solving a 3TQ.

It cannot be scored for stating the roots directly via use of a calculator which is dM0A0

Note that e.g. $27k^4-33k^2-4=0\Rightarrow k^2=\frac{4}{3}$ scores dM0A0

$$27k^4-33k^2-4=0\Rightarrow k^2=\frac{-b\pm\sqrt{b^2-4ac}}{2a}\Rightarrow k^2=\frac{4}{3}\text{ scores dM0A0}$$

$$27k^4-33k^2-4=0\Rightarrow\left(k^2+\frac{1}{9}\right)\left(k^2-\frac{4}{3}\right)=0\Rightarrow k^2=\ldots\text{ scores dM0A0}$$

$$27k^4-33k^2-4=0\Rightarrow 27\left(k^2+\frac{1}{9}\right)\left(k^2-\frac{4}{3}\right)=0\Rightarrow k^2=\ldots\text{ scores dM1}$$

$$27k^4-33k^2-4=0\Rightarrow\left(k^2=\right)\frac{33\pm\sqrt{33^2-4(-4)(27)}}{2\times 27}\Rightarrow k^2=\frac{4}{3}\text{ scores dM1}$$

Note that some candidates will use a substitution to solve their $27k^4-33k^2-4=0$ e.g. $x=k^2\Rightarrow 27x^2-33x-4=0\Rightarrow x=\ldots$

In such cases, the dM1 mark can be scored for solving their 3TQ by any acceptable correct **non-calculator** method to obtain a **positive** value for x.

Do **not** condone e.g. $27k^4-33k^2-4=0$, using $k=k^2\Rightarrow(9k+1)(3k-4)=0\Rightarrow k=\frac{4}{3}$ unless recovered.

A1: Deduces the correct **exact** values. Both are required. Allow any equivalent exact values e.g. $k=\pm\frac{2\sqrt{3}}{3},\ \pm\sqrt{\frac{4}{3}}$. Apply isw once the correct exact answers are seen (e.g. if they change them to decimals) but if any other values are given and are not rejected or either of the correct ones are clearly rejected, score A0

Condone mislabelling e.g. $x=\pm\frac{2\sqrt{3}}{3}$ and just look for both correct exact values.

Note that M1dM0A1 is **not** a possible mark profile as the A mark must follow **both** M marks.

In 4(b) there may be alternative valid approaches that do not directly use the result from (a) e.g. these have been seen:

$$u_3-5u_2=11\Rightarrow 3\left(3k^2-1\right)^2-1-5\left(3k^2-1\right)=11$$

$$\Rightarrow 3\left(3k^2-1\right)^2-5\left(3k^2-1\right)-12=0$$

$$x=3k^2-1 \Rightarrow 3x^2-5x-12=0 \Rightarrow (3x+4)(x-3)=0$$

$$x=3 \Rightarrow 3k^2-1=3 \Rightarrow 3k^2=4 \Rightarrow k=\pm\frac{2}{\sqrt{3}}$$

or

$$u_3-5u_2=11 \Rightarrow 3u_2{}^2-1-5u_2=11$$

$$\Rightarrow 3u_2{}^2-5u_2-12=0 \Rightarrow (3u_2+4)(u_2-3)=0$$

$$u_2=3 \Rightarrow 3k^2-1=3 \Rightarrow 3k^2=4 \Rightarrow k=\pm\frac{2}{\sqrt{3}}$$

These can be marked in the same way e.g.

M1: For using $u_3-5u_2=11$ to obtain a 3TQ in u_2 or their u_2 in terms of *k*.

dM1: For solving their 3TQ by any acceptable correct non-calculator means to obtain a positive value for u_2 or their u_2

A1: As in the main scheme.