Question	Scheme	Marks	AOs
10(a)	$4x^2-x+1\equiv A(1-x)(1-2x)+B(1-2x)+C(1-x)$ $\Rightarrow B=...$ or $C=...$	M1	1.1b
	$A=2$	B1	2.2a
	$B=-4$ or $C=3$	A1	1.1b
	$B=-4$ and $C=3$	A1	1.1b
		(4)	

Notes

M1: Multiplies through by $(1-x)(1-2x)$ to reach a correct identity condoning **one** slip in copying $4x^2-x+1$ and finds a value for at least one of B or C either by substituting values or comparing coefficients and solving simultaneous equations.

Note a correct identity may be implied by $\dfrac{4x^2-x+1}{(1-x)(1-2x)}\equiv\dfrac{A(1-x)(1-2x)+B(1-2x)+C(1-x)}{(1-x)(1-2x)}$

Note substitution gives e.g. $x=1\to 4=-B\to B=-4,\ x=\dfrac{1}{2}\to\dfrac{3}{2}=\dfrac{1}{2}C\to C=3$

Note comparing coefficients gives: $x^2:4=2A;\ x:-1=-3A-2B-C;$ constant $:1=A+B+C$

Note that $4x^2-x+1\equiv B(1-2x)+C(1-x)\to B=...$ or $C=...$

and e.g. $4x^2-x+1\equiv A+B(1-2x)+C(1-x)\to B=...$ or $C=...$ both score M0

Note that a correct identity may be implied by their work e.g. if they omit the lhs but compare coefficients correctly.

B1: Deduces $A=2$ anywhere in their solution.

A1: Either B or C correct from a correct identity.

A1: Both B and C correct from a correct identity.

Note that the values may be seen when they form the identity if they are not seen explicitly first.

Do not allow e.g. $\dfrac{6}{2}$ for 3 unless $\dfrac{6}{2}$ becomes 3 later then it can be recovered.

Apply isw once the correct values are seen e.g. if they subsequently write down the identity with the values in the wrong places.

Correct values of *B* and *C* from <u>no</u> working scores M1A1A1

	(a) Alternative by long division:		
	$\dfrac{4x^2-x+1}{(1-x)(1-2x)}\equiv 2+\dfrac{5x-1}{(1-x)(1-2x)}$		
	$\dfrac{5x-1}{(1-x)(1-2x)}\equiv\dfrac{B}{1-x}+\dfrac{C}{1-2x}\Rightarrow 5x-1\equiv B(1-2x)+C(1-x)$ $\Rightarrow B=...$ or $C=...$	M1	1.1b
	$A=2$	B1	2.2a
	$B=-4$ or $C=3$	A1	1.1b
	$B=-4$ and $C=3$	A1	1.1b

M1: Attempts to divide $4x^2-x+1$ by $(1-x)(1-2x)$ to obtain $\alpha+\dfrac{\beta x+\gamma}{(1-x)(1-2x)},\ \alpha,\beta,\gamma\neq 0$ and then uses a correct identity $\beta x+\gamma\equiv B(1-2x)+C(1-x)$ for their $\beta x+\gamma$ and finds a value for at least one of B or C either by substituting values or comparing coefficients and solving simultaneous equations. Condone a slip on the lhs.

Note a correct identity may be implied by e.g. $\dfrac{5x-1}{(1-x)(1-2x)}\equiv\dfrac{B(1-2x)+C(1-x)}{(1-x)(1-2x)}$

Allow use of other letters e.g. they may use A and B here for B and C and condone this for the A marks if this mislabelling persists.

Note substitution gives e.g. $x=1\rightarrow 4=-B\rightarrow B=-4,\ x=\frac{1}{2}\rightarrow\frac{3}{2}=\frac{1}{2}C\rightarrow C=3$

Note comparing coefficients gives: $x:5=-2B-C;$ $\text{constant}:-1=B+C$

Note that a correct identity may be implied by their work e.g. if they omit the lhs but compare coefficients correctly.

B1: Deduces A = 2 anywhere in their solution which may appear in their quotient.

A1: Either B or C correct from a correct identity and correct remainder.

A1: Both B and C correct from a correct identity and correct remainder.
Note that the values may be seen when they form the identity if they are not seen explicitly first.

Do not allow e.g. $\frac{6}{2}$ for 3 unless $\frac{6}{2}$ becomes 3 later then it can be recovered.

Apply isw once the correct values are seen e.g. if they subsequently write down the identity with the values in the wrong places.

Correct values of *B* and *C* from <u>no</u> working scores M1A1A1

<u>Special Case: Beware of correct values following an incorrect method:</u>

$$\frac{4x^2-x+1}{(1-x)(1-2x)}\equiv 2+\frac{5x-1}{(1-x)(1-2x)}$$

$$\frac{4x^2-x+1}{(1-x)(1-2x)}\equiv\frac{B}{1-x}+\frac{C}{1-2x}\Rightarrow 4x^2-x+1\equiv B(1-2x)+C(1-x)\rightarrow B=-4,\ C=3$$

<u>This scores a maximum of M0B1A0A0 in (a)</u>

<u>Part (b) can be marked in the normal way but withhold the final mark if they obtain the correct answer as it follows incorrect work.</u>

<u>Note that there may be some cases where:</u>

1. **Candidates initially divide $4x^2-x+1$ by (1 – *x*) or (1 – 2*x*) first before setting up an identity.**
2. **Candidates substitute 3 values of *x* (other than 1 or $\frac{1}{2}$) into the given identity to produce 3 equations in 3 unknowns and then solve simultaneously to find *A, B* and *C*.**

10(b)	$\dfrac{4x^2-x+1}{(1-x)(1-2x)}=2-4(1-x)^{-1}+3(1-2x)^{-1}$	M1	3.1a
	$(1-x)^{-1}=1+x+x^2$ **or** $(1-2x)^{-1}=1+2x+4x^2$	A1	1.1b
	$(1-x)^{-1}=1+x+x^2$ **and** $(1-2x)^{-1}=1+2x+4x^2$	A1	1.1b
	$\dfrac{4x^2-x+1}{(1-x)(1-2x)}=2-4(1+x+x^2)+3(1+2x+4x^2)=...$	**d**M1	2.1
	$=1+2x+8x^2$	A1	1.1b
		(5)	

M1: Recognises the requirement to write the denominator of **either** fraction as a power of −1 e.g. $(1-x)^{-1}$ or $(1-2x)^{-1}$ so that appropriate expansions can be found.

A1: Correct expansion for $(1-x)^{-1}$ **or** $(1-2x)^{-1}$ up to at least the term in x^2.

Any extra terms can be ignored.

Allow unsimplified e.g. $(1-x)^{-1}=1+(-1)(-x)+\dfrac{-1\times(-1-1)}{2}(\pm x)^2+...$

or e.g. $(1-2x)^{-1}=1+(-1)(-2x)+\dfrac{-1\times(-1-1)}{2}(\pm 2x)^2+...$

You may see attempts using direct expansions e.g.

$(1-x)^{-1}=1^{-1}+(-1)(1^{-2})(-x)+\dfrac{-1\times(-1-1)}{2}(1^{-3})(\pm x)^2+...$

or $(1-2x)^{-1}=1^{-1}+(-1)(1^{-2})(-2x)+\dfrac{-1\times(-1-1)}{2}(1^{-3})(\pm 2x)^2+...$

where you can condone missing powers on the 1's

Do not condone missing brackets unless they are implied by subsequent work.

The correct expansions may also be implied if the expansion of $(1-x)^{-1}$ has been combined with their "*B*" or if the expansion of $(1-2x)^{-1}$ has been combined with their "*C*" e.g. for correct *B* and *C* this is $-4-4x-4x^2$ and $3+6x+12x^2$ respectively.

A1: Correct expansions for $(1-x)^{-1}$ **and** $(1-2x)^{-1}$ up to at least the term in x^2.

Any extra terms can be ignored.

May be unsimplified and may be combined with "*B*" and/or "*C*" as for the first A mark.

dM1: A complete method to find the expansion required condoning the failure to include their *A* in the expansion but if they do include their *A*, it must be used correctly.

This is **dependent on the first method** mark and requires an attempt to expand both $(1-x)^{-1}$ **and** $(1-2x)^{-1}$ up to at least x^2 and an attempt to collect terms.

Condone 'poor' attempts to expand but there must be attempts at $(1-x)^{-1}$ **and** $(1-2x)^{-1}$ up to at least x^2

Condone slips when they combine their expansions with their *B* and *C* and condone if they "forget" to apply their *B* and *C* and condone the use of the letters *A*, *B* and *C*.

A1: Correct expression. If any extra terms are found they can be ignored.

Apply isw once a correct expression is seen so condone e.g. $=1+2x+8x^2=0$

Allow the terms to be listed and allow them to appear in any order.

10(b) Using Maclaurin series:

10(b)	$\mathrm{f}(x)=2-\frac{4}{(1-x)}+\frac{3}{(1-2x)} \Rightarrow \mathrm{f}'(x)=-4(1-x)^{-2}+6(1-2x)^{-2}$	M1 A1	3.1a 1.1b
	$\Rightarrow \mathrm{f}''(x)=-8(1-x)^{-3}+24(1-2x)^{-3}$	A1	1.1b
	$\mathrm{f}(0)=1,\ \mathrm{f}'(0)=2,\ \mathrm{f}''(0)=16$ $\mathrm{f}(x)=1+2x+\frac{16}{2}x^2+...$	**d**M1	2.1
	$=1+2x+8x^2$	A1	1.1b
		(5)	

Notes

M1: Differentiates and obtains at least $...(1-x)^{-2}$ or $(1-2x)^{-2}$

A1: Correct first derivative. Follow through their *B* and *C*.

A1: Correct second derivative. Follow through their *B* and *C*.

dM1: Correct attempt at the Maclaurin series.

A1: As main scheme.

We will condone attempts to expand $\frac{4x^2-x+1}{(1-x)(1-2x)}$ **as** $(4x^2-x+1)(1-x)^{-1}(1-2x)^{-1}$

and mark as follows.

10(b)	$\frac{4x^2-x+1}{(1-x)(1-2x)}=(4x^2-x+1)(1-x)^{-1}(1-2x)^{-1}$	M1	3.1a
	$(1-x)^{-1}=1+x+x^2$ **or** $(1-2x)^{-1}=1+2x+4x^2$	A1	1.1b
	$(1-x)^{-1}=1+x+x^2$ **and** $(1-2x)^{-1}=1+2x+4x^2$	A1	1.1b
	$(4x^2-x+1)(1+x+x^2)(1+2x+4x^2)=...$	**d**M1	2.1
	$=1+2x+8x^2$	A1	1.1b
		(5)	

Notes

M1: As main scheme.

A1: As main scheme.

A1: As main scheme.

dM1: Expands all 3 brackets including an attempt to expand both $(1-x)^{-1}$ **and** $(1-2x)^{-1}$ up to at least x^2 and an attempt to collect terms.

A1: As main scheme.

10(c)	$\lvert x\rvert < \frac{1}{2}$ or e.g. $-\frac{1}{2} < x < \frac{1}{2}$	B1	2.5
		(1)	
			(10 marks)

Notes

B1: Correct range using the correct notation in terms of x.

Other acceptable answers are: $\left(-\frac{1}{2}, \frac{1}{2}\right)$, "$x > -\frac{1}{2}$ and $x < \frac{1}{2}$" "$x > -\frac{1}{2} \cap x < \frac{1}{2}$"

Condone e.g. $-\frac{1}{2} < \lvert x\rvert < \frac{1}{2}$

You can ignore superfluous bracketing e.g. $\left\{-\frac{1}{2} < x < \frac{1}{2}\right\}$ or $\left(-\frac{1}{2} < x < \frac{1}{2}\right)$

Do **not** accept e.g. "$x > -\frac{1}{2}$ or $x < \frac{1}{2}$", "$x > -\frac{1}{2}, x < \frac{1}{2}$", "$x > -\frac{1}{2} \cup x < \frac{1}{2}$",

$\lvert x\rvert < -\frac{1}{2}$, $\lvert -x\rvert < \frac{1}{2}$

Do **not** condone non-strict inequalities.

If more than one answer is given and an answer is not clearly selected mark their final answer.