Question	Scheme	Marks	AOs
11(a)	$u=\sqrt{5x-2} \Rightarrow u^2=5x-2 \Rightarrow x=\frac{u^2+2}{5} \Rightarrow \frac{dx}{du}=\frac{2u}{5}$ or e.g. $u=\sqrt{5x-2} \Rightarrow \frac{du}{dx}=\frac{1}{2}(5x-2)^{-\frac{1}{2}}\times 5$	B1	1.1b
	$\int \frac{1}{3+\sqrt{5x-2}}\,dx=\int \frac{1}{3+u}\times\frac{2u}{5}(du)$ or e.g. $\int \frac{1}{3+\sqrt{5x-2}}\,dx=\int \frac{1}{3+u}\times\frac{2}{5}\sqrt{5x-2}(du)=\int \frac{1}{3+u}\times\frac{2u}{5}(du)$	M1	1.1b
	$x=\frac{3}{5}\Rightarrow u=\sqrt{3-2}=1,\ x=\frac{27}{5}\Rightarrow u=\sqrt{27-2}=5$	M1	1.1b
	$\int_{\frac{3}{5}}^{\frac{27}{5}} \frac{1}{3+\sqrt{5x-2}}\,dx=\frac{2}{5}\int_1^5 \frac{u}{3+u}\,du$	A1	2.1
		(4)	

Notes

B1: Correct equation connecting dx and du. See main scheme for most likely examples but allow equivalents e.g. $5dx=2u\,du$ or $du=\frac{5}{2}(5x-2)^{-\frac{1}{2}}dx$ and award once a correct equation is seen. Allow a "dx" or "du" to be missing if it is subsequently recovered.

M1: For this mark they must be using one of:

$\frac{dx}{du}=...u$ **or** $dx=...u\,du$ **or** $\frac{du}{dx}=...(5x-2)^{-\frac{1}{2}}$ **or** $du=...(5x-2)^{-\frac{1}{2}}dx$ **or** $\frac{du}{dx}=\frac{...}{u}$

where "…" is a non-zero constant.

If they start with an incorrect form but rearrange to fortuitously obtain one of the required forms, do not allow this mark unless it is a clear restart.

e.g. do not allow $\frac{du}{dx}=\frac{1}{2}(5x-2)^{\frac{1}{2}}\times 5$ leading to $du=...(5x-2)^{-\frac{1}{2}}dx$

Attempts to substitute, fully replacing each of $\frac{1}{3+\sqrt{5x-2}}$ and dx to reach an expression in one of the correct forms in terms of u only.

There is no need to replace the limits for this mark.

Condone the omission of the "du" and/or the integral sign. It requires one of:

$$\frac{dx}{du}=...u \text{ or } dx=...u\,du \to \int \frac{1}{3+\sqrt{5x-2}}\,dx=\int \frac{1}{3+u}\times ...u(du) \text{ oe}$$

or

$$\frac{du}{dx}=...(5x-2)^{-\frac{1}{2}} \text{ or } du=...(5x-2)^{-\frac{1}{2}}dx$$

$$\to \int \frac{1}{3+\sqrt{5x-2}}\,dx=\int \frac{1}{3+u}\times\frac{(du)}{...(5x-2)^{-\frac{1}{2}}}=\int \frac{1}{3+u}\times\frac{(du)}{...u^{-1}} \text{ oe}$$

or $$\int \frac{1}{3+\sqrt{5x-2}}\,dx=\int \frac{1}{3+u}\times ...(5x-2)^{\frac{1}{2}}(du)=\int \frac{1}{3+u}\times ...u(du) \text{ oe}$$

M1:	Attempts to use both $x=\frac{3}{5}$ and $x=\frac{27}{5}$ with $u=\sqrt{5x-2}$ to find values for u. May be implied by at least one correct value but both limits must be attempted.
A1:	All correct with correct limits and correct k with no errors and the k factored outside the integral. The "du" and the integral sign must be present but may have been omitted along the way but they must have been seen at least once together before the final answer. The limits can be seen as part of the integral or stated separately.

Note that we will condone as a **special case** for full marks in (a) a minimum response of:

$$u=\sqrt{5x-2}\Rightarrow x=\frac{u^2+2}{5}\Rightarrow\frac{\mathrm{d}x}{\mathrm{d}u}=\frac{2u}{5}\Rightarrow\int\frac{1}{3+\sqrt{5x-2}}\,\mathrm{d}x=\frac{2}{5}\int_1^5\frac{u}{3+u}\,\mathrm{d}u$$

11(b)	$\int\frac{u}{3+u}\,\mathrm{d}u=\int\frac{3+u-3}{3+u}\,\mathrm{d}u=\int 1-\frac{3}{3+u}\,\mathrm{d}u$	M1	3.1a
	$\int 1-\frac{3}{3+u}\,\mathrm{d}u=u-3\ln(3+u)(+c)$	dM1	2.1
	$\left("\frac{2}{5}"\right)\int\frac{u}{3+u}\,\mathrm{d}u=\left("\frac{2}{5}"\right)(u-3\ln(3+u))(+c)$	A1ft	1.1b
	$\left[\frac{2}{5}(u-3\ln(3+u))\right]_1^5=\frac{2}{5}(5-3\ln 8-1+3\ln 4)=\frac{8}{5}-\frac{6}{5}\ln 2$	A1	2.1
		(4)	
			(8 marks)

M1:	Adopts an appropriate strategy by writing the fraction as $\alpha+\frac{\beta}{3+u},\ \alpha,\beta\neq 0$
dM1:	Integrates to the form $...u+...\ln(3+u)$ or e.g. $...u+...\ln\lvert 3+u\rvert$ Note that $...u+...\ln\lambda(3+u)$ where λ is a positive constant or e.g. $...u+...\ln\lvert\lambda(3+u)\rvert$ where λ is a constant are also acceptable. Condone missing brackets for this mark e.g. $...u+...\ln 3+u$
A1ft:	Correct integration. Follow through their k from part (a) or the letter k and condone its omission for this mark. (Condone lack of limits here) Do not condone missing brackets for this mark e.g. $u-3\ln 3+u$ unless the missing brackets are implied by subsequent work e.g. their use of their limits.
A1:	Fully correct work proceeding to $\frac{8}{5}-\frac{6}{5}\ln 2$ or $\frac{8}{5}-\frac{6}{5}\ln\lvert 2\rvert$ but not e.g. $\frac{2}{5}(4-3\ln 2)$ unless the correct version was seen previously. Allow correct work leading to $A=\frac{8}{5},\ B=-\frac{6}{5}$ Allow any equivalent exact forms of $\frac{8}{5}$ and/or $\frac{6}{5}$

Note that it is possible to do part (b) using another substitution e.g.

$$y = 3+u \to \int \frac{u}{3+u}\,\mathrm{d}u = \int \frac{y-3}{y}\,\mathrm{d}y = \int 1-\frac{3}{y}\,\mathrm{d}y$$

$$\int 1-\frac{3}{y}\,\mathrm{d}y = y - 3\ln y\,(+c)$$

$$\frac{2}{5}\int_1^5 \frac{u}{3+u}\,\mathrm{d}u = \frac{2}{5}\left[y-3\ln y\right]_4^8 = \frac{2}{5}\left(8-3\ln 8-4+3\ln 4\right) = \frac{8}{5}-\frac{6}{5}\ln 2$$

This marks in a similar way to the main scheme:

M1: Adopts an appropriate strategy by writing the fraction as $\alpha+\frac{\beta}{y}$, $\alpha,\beta \neq 0$

dM1: Integrates to the form $...y+...\ln y$

A1ft: Correct integration. Follow through their *k* from part (a) and condone its omission for this mark. (Condone lack of limits here)

A1: Fully correct work proceeding to $\frac{8}{5}-\frac{6}{5}\ln 2$ or $\frac{8}{5}-\frac{6}{5}\ln|2|$ via the correctly changed limits or reverses the substitution and uses the limits 1 and 5. Do not allow e.g. $\frac{2}{5}(4-3\ln 2)$ unless the correct version was seen previously. Allow correct work leading to $A=\frac{8}{5}$, $B=-\frac{6}{5}$ Allow any equivalent exact forms of $\frac{8}{5}$ and/or $\frac{6}{5}$

It is also possible to use integration by parts twice which may include another substitution:

$$\int \frac{u}{3+u}\,\mathrm{d}u = u\ln(3+u) - \int \ln(3+u)\,\mathrm{d}u$$

$$= u\ln(3+u) - \left((3+u)\ln(3+u)-(3+u)\right)(+c)$$

$$\frac{2}{5}\left[u\ln(3+u)-\left((3+u)\ln(3+u)-(3+u)\right)\right]_1^5 = \frac{2}{5}\left(5\ln 8-(8\ln 8-8)-\left(\ln 4-(4\ln 4-4)\right)\right) = \frac{8}{5}-\frac{6}{5}\ln 2$$

Score **M1dM1** simultaneously when they reach the form:

$$\left(\frac{2}{5}\right)\int \frac{u}{3+u}\,\mathrm{d}u = \left(\frac{2}{5}\right)\left(u\ln(3+u)-\left((3+u)\ln(3+u)-(3+u)\right)\right)(+c)$$

oe e.g. $\left(\frac{2}{5}\right)\left(u\ln(3+u)-\left((3+u)\ln(3+u)-u\right)\right)(+c)$

The method must be complete and correct with sign or coefficient errors only and they must reach this form **or equivalent** and **A1ft** if correct with or without the "$\frac{2}{5}$" and allow the letter *k*.

A1: $\frac{8}{5}-\frac{6}{5}\ln 2$ As main scheme