Question	Scheme	Marks	AOs
12(a)	$t=\frac{\pi}{4}\Rightarrow x=1,\ y=\frac{1}{\sqrt{2}}$ or $\left(1,\ \frac{1}{\sqrt{2}}\right)$	B1	1.1b
		(1)	

Notes

B1: Correct value for x and correct **exact** value for y. Allow exact equivalents for y e.g. $\frac{\sqrt{2}}{2}$

Allow as a coordinate pair or as $x=\ldots,\ y=\ldots$

For a coordinate pair condone missing brackets e.g. $1,\ \frac{1}{\sqrt{2}}$ or unusual notation e.g. $\left(1;\ \frac{1}{\sqrt{2}}\right)$ or as a column vector e.g. $\begin{pmatrix}1\\ \frac{1}{\sqrt{2}}\end{pmatrix}$ but do **not** allow coordinates the wrong way round unless seen correctly as $x=1,\ y=\frac{1}{\sqrt{2}}$ previously and then apply isw.

Question	Scheme	Marks	AOs
12(b)	$\left(\frac{\mathrm{d}y}{\mathrm{d}t}=\right)\cos t\sin 2t+2\sin t\cos 2t$ or e.g. $y=\sin t\sin 2t=2\sin^2 t\cos t\Rightarrow\left(\frac{\mathrm{d}y}{\mathrm{d}t}=\right)4\sin t\cos^2 t-2\sin^3 t$ or e.g. $y=\sin t\sin 2t=2\sin^2 t\cos t=2\left(1-\cos^2 t\right)\cos t=2\cos t-2\cos^3 t$ $\Rightarrow\left(\frac{\mathrm{d}y}{\mathrm{d}t}=\right)-2\sin t+6\cos^2 t\sin t$	M1	1.1b
	$\left(\frac{\mathrm{d}x}{\mathrm{d}t}=\right)2\sin 2t$ or e.g. $\left(\frac{\mathrm{d}x}{\mathrm{d}t}=\right)4\sin t\cos t$	M1	1.1b
	$\left(\frac{\mathrm{d}y}{\mathrm{d}t}=\right)\cos t\sin 2t+2\sin t\cos 2t$ oe **and** $\left(\frac{\mathrm{d}x}{\mathrm{d}t}=\right)2\sin 2t$ oe	A1	1.1b
	$t=\frac{\pi}{4}\Rightarrow\left(\pm\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\pm\frac{\cos\frac{\pi}{4}\sin\frac{\pi}{2}+2\sin\frac{\pi}{4}\cos\frac{\pi}{2}}{2\sin\frac{\pi}{2}}=\ldots$ **or** $t=\frac{\pi}{4}\Rightarrow\left(\pm\frac{\mathrm{d}x}{\mathrm{d}y}=\right)\pm\frac{2\sin\frac{\pi}{2}}{\cos\frac{\pi}{4}\sin\frac{\pi}{2}+2\sin\frac{\pi}{4}\cos\frac{\pi}{2}}=\ldots$	M1	1.1b
	$y-\frac{1}{\sqrt{2}}=-2\sqrt{2}\left(x-1\right)$ or e.g. $\frac{y-\frac{1}{\sqrt{2}}}{x-1}=-2\sqrt{2}$ or e.g.	dM1	2.1

	$y=-2\sqrt{2}x+c,\ \dfrac{1}{\sqrt{2}}=-2\sqrt{2}\times 1+c \Rightarrow c=...$		
	$4x+\sqrt{2}y=5$	A1	2.2a
		(6)	

Notes

<u>They must use parametric differentiation in this part so do not allow attempts at e.g. Cartesian forms.</u>

M1: Differentiates the y parameter to obtain $\pm...\cos t\sin 2t \pm ...\sin t\cos 2t$ or equivalent where "…" is a constant.
If any identities are used before differentiating then they must be correct and the subsequent differentiation of the correct form with sign and/or coefficient slips only.
e.g. replaces $\sin 2t$ with $2\sin t\cos t$ and obtains $\pm...\cos^2 t\sin t \pm ...\sin^3 t$ oe
or e.g. replaces $\sin 2t$ with $2\sin t\cos t$ and $\sin^2 t$ with $1-\cos^2 t$ and obtains $\pm...\sin t \pm ...\cos^2 t\sin t$ oe where "…" is a constant.
If the product rule is quoted, it must be correct.

M1: Differentiates the x parameter to obtain $\pm...\sin 2t$ or equivalent e.g. $\pm...\sin t\cos t$ where "…" is a constant. If they use an identity first, it must be correct.

A1: Both derivatives correct $\left(\frac{dy}{dt}=\right)\cos t\sin 2t+2\sin t\cos 2t$ oe **and** $\left(\frac{dx}{dt}=\right)2\sin 2t$ oe
Apply isw once the correct derivatives are seen.

M1: Divides their differentiated parameters either way round and substitutes $t=\dfrac{\pi}{4}$ and proceeds to a non-zero value for $\pm\dfrac{dy}{dx}$ or $\pm\dfrac{dx}{dy}$ following an attempt to differentiate **both** parameters however poor.
You can ignore any labelling.
May be implied by e.g.
$$\frac{dx}{dt}=2\sin 2t,\ \frac{dy}{dt}=\cos t\sin 2t+2\sin t\cos 2t, t=\frac{\pi}{4}\Rightarrow\frac{dx}{dt}=2, \frac{dy}{dt}=\frac{1}{\sqrt{2}}\Rightarrow\frac{dy}{dx}=\frac{\sqrt{2}}{4}$$
If no substitution is seen and just a value is written down you may need to check but if they refer to $t=\dfrac{\pi}{4}$ and a value is written down you do not need to check and M1 can be awarded. Attempts to use e.g. $\dfrac{dy}{dx}=\dfrac{y}{x}$ scores M0

dM1: For a **<u>correct method</u>** at the normal equation using
- their x and y from (a)
- the negative reciprocal of their $\dfrac{dy}{dt}\div$ their $\dfrac{dx}{dt}$ at $t=\dfrac{\pi}{4}$

If they use $y = mx + c$ they must reach as far as obtaining a value for c.
Depends on the previous method mark.

A1: Correct equation from correct work in the form required.
Allow $\sqrt{2}y+4x=5$ or $5=4x+\sqrt{2}y$ or $5=\sqrt{2}y+4x$

Note that some candidates are obtaining the correct answer to part (b) fortuitously e.g. following incorrect differentiation that still leads to the correct gradient. In such cases, the A marks in part (c) should be withheld as this means correct answers are following incorrect work.

12(c)	$\begin{matrix}"4"x+\sqrt{2}y="5" \\ 2y^2=x^2(2-x)\end{matrix} \Rightarrow 2\left(\dfrac{"5"-"4"x}{\sqrt{2}}\right)^2=x^2(2-x)$ or e.g. $"4x"+\sqrt{2}y="5" \Rightarrow \sqrt{2}y="5"-"4"x$ $\Rightarrow 2y^2=("5"-"4"x)^2 \Rightarrow ("5"-"4"x)^2=x^2(2-x)$	M1	3.1a
	$\Rightarrow x^3+14x^2-40x+25=0$	A1	1.1b
	$x^3+14x^2-40x+25=0 \Rightarrow (x-"1")(x^2+15x-25)=0$ $x^2+15x-25=0 \Rightarrow x=...$	M1	2.1
	$x=\dfrac{-15+5\sqrt{13}}{2}$ or $x=-\dfrac{15}{2}+\dfrac{5}{2}\sqrt{13}$ or $x=-7.5+2.5\sqrt{13}$	A1	2.3
		(4)	
			(11 marks)

M1: Uses their straight line equation from part (b) to obtain *y* in terms of *x* and substitutes into the equation for *C* to obtain an equation in *x* only.
Condone 'poor' algebra or e.g. miscopies of the equation for *C* or their straight line as long as the intention is clear and an equation in one variable is obtained.
Their straight line equation does not need to have the form $ax+\sqrt{2}y=b$ but must be equivalent i.e. $px+qy=r$ with *p*, *q*, *r* non-zero.
Allow slips in rearranging their equation from part (b) as long as the intention is clear.
Allow the use of $ax+\sqrt{2}y=b$ for *l*, i.e. with the letters *a* and *b* and allow a "made-up" *a* and *b*.

A1: Correct cubic equation in *x* with terms collected and all on one side.
Allow any multiple of this equation and the "= 0" may be implied by their attempt to solve.

M1: Uses the fact that (x – 1), where "1" is their *x* coordinate of *P*, is a factor of their cubic expression which must be of the form $\alpha x^3+\beta x^2+\gamma x+\delta \quad \alpha,\beta,\gamma,\delta \neq 0$ using inspection or long division or e.g. expanding $(x-"1")(Ax^2+Bx+C)$ and comparing coefficients in an attempt to establish a 3 term quadratic factor **and** then solves the resulting 3TQ by a correct method **including a calculator** to obtain at least one exact and real value for *x*. They must have numeric values for *a* and *b* (for *l*) for this mark and allow if they are "made-up" values.
Condone attempts by e.g. long division that include a remainder when dividing by (x – "1")
Do not condone attempts to solve an equation that does not have real roots.
Do not be too concerned with how the quadratic factor is achieved but do not allow methods that are clearly not using the factor of (x – "1") e.g. by dividing their cubic by *x* or differentiating.
Do **not** allow attempts to solve their **cubic** on a calculator without sight of a 3TQ first e.g. $x^3+14x^2-40x+25=0 \Rightarrow x=1, \dfrac{-15+5\sqrt{13}}{2}$ is M0 but condone e.g.

$x^3+14x^2-40x+25=0 \to x^2+15x-25 \to \dfrac{-15+5\sqrt{13}}{2}$ i.e. if the (x – “1”) is not seen explicitly so the **correct** quadratic for their cubic can imply this mark.

A1: Correct exact value and no others but ignore x = “1”. Ignore any attempts to find the y coordinate.

(c) Alternative eliminating *x*:

12(c)	$\begin{array}{l}4x+\sqrt{2}y=5\\2y^2=x^2(2-x)\end{array} \Rightarrow 2y^2=\left(\dfrac{5-\sqrt{2}y}{4}\right)^2\left(2-\dfrac{5-\sqrt{2}y}{4}\right)$	M1	3.1a
	$\Rightarrow 2\sqrt{2}y^3-142y^2-5\sqrt{2}y+75=0$	A1	1.1b
	$2\sqrt{2}y^3-142y^2-5\sqrt{2}y+75=0$ $\Rightarrow\left(y-\text{"}\dfrac{1}{\sqrt{2}}\text{"}\right)\left(2\sqrt{2}y^2-140y-75\sqrt{2}\right)=0$ $2\sqrt{2}y^2-140y-75\sqrt{2}=0 \Rightarrow y=...\left(\dfrac{35\sqrt{2}\pm10\sqrt{26}}{2}\right)$ $4x+\sqrt{2}y=5 \Rightarrow 4x+\sqrt{2}\left(\dfrac{35\sqrt{2}\pm10\sqrt{26}}{2}\right)=5 \Rightarrow x=...$	M1	2.1
	$x=\dfrac{-15+5\sqrt{13}}{2}$ or $x=-\dfrac{15}{2}+\dfrac{5}{2}\sqrt{13}$ or $x=-7.5+2.5\sqrt{13}$	A1	2.3
		(4)	

(11 marks)

M1: Uses their straight line equation from part (b) to obtain x in terms of y and substitutes into the equation for C to obtain an equation in y only.
Condone ‘poor’ algebra or e.g. miscopies of the equation for C or their straight line as long as the intention is clear and an equation in one variable is obtained.
Their straight line equation does not need to have the form $ax+\sqrt{2}y=b$ but must be equivalent e.g. $px+qy=r$ with $p,\ q,\ r$ non-zero.
Allow slips in rearranging their equation from part (b) as long as the intention is clear.
Allow the use of $ax+\sqrt{2}y=b$ for l, i.e. with the letters a and b and allow a “made-up” a and b.

A1: Correct cubic equation in y with terms collected and all on one side.
Allow any multiple of this equation and the “= 0” may be implied by their attempt to solve.

M1: Uses the fact that $\left(y-\frac{1}{\sqrt{2}}\right)$ where “$\frac{1}{\sqrt{2}}$” is their y coordinate of P, is a factor of their cubic expression which must be of the form $\alpha y^3+\beta y^2+\gamma y+\delta$ $\alpha,\beta,\gamma,\delta\neq0$ using inspection or long division in an attempt to establish the 3 term quadratic factor **and** then solves the resulting 3TQ by a correct method **including a calculator** to obtain at least one exact and real value for y and then uses their equation for l to find an exact value for x. Do not condone attempts to solve an equation that does not have real roots.
They must have numeric values for a and b (for l) for this mark and allow if they are “made-up” values.
Condone attempts by e.g. long division that include a remainder when dividing

	by $\left(y-"\frac{1}{\sqrt{2}}"\right)$ and you do not need to check the long division/inspection as long as a 3 term quadratic factor is obtained. Do not be too concerned with how the quadratic factor is achieved but do not allow methods that are clearly not using the factor of $\left(y-"\frac{1}{\sqrt{2}}"\right)$ e.g. by dividing their cubic by y or differentiating. Do **not** allow attempts to solve their **cubic** on a calculator e.g. $2\sqrt{2}y^3-142y^2-5\sqrt{2}y+75=0\Rightarrow y=...\Rightarrow x=...$
A1:	Correct exact value and no others but ignore $x=$ "1".

(c) Alternative using parametric form:

12(c)	$4x+\sqrt{2}y=5\Rightarrow 4(1-\cos 2t)+\sqrt{2}\sin t\sin 2t=5$ $\Rightarrow 4\left(1-\left(2\cos^2 t-1\right)\right)+2\sqrt{2}\sin^2 t\cos t=5$ $\Rightarrow 4-8\cos^2 t+4+2\sqrt{2}\left(1-\cos^2 t\right)\cos t=5$	M1	3.1a
	$\Rightarrow 2\sqrt{2}\cos^3 t+8\cos^2 t-2\sqrt{2}\cos t-3=0$	A1	1.1b
	$2\sqrt{2}\cos^3 t+8\cos^2 t-2\sqrt{2}\cos t-3=0$ $\left(\cos t-\frac{1}{\sqrt{2}}\right)\left(2\sqrt{2}\cos^2 t+10\cos t+3\sqrt{2}\right)=0$ $2\sqrt{2}\cos^2 t+10\cos t+3\sqrt{2}=0\Rightarrow \cos t=...\left(\frac{-5\sqrt{2}\pm\sqrt{26}}{4}\right)$ $x=1-\cos 2t=1-\left(2\cos^2 t-1\right)=2-2\left(\frac{-5\sqrt{2}\pm\sqrt{26}}{4}\right)^2$	M1	2.1
	$x=\frac{-15+5\sqrt{13}}{2}$ or $x=-\frac{15}{2}+\frac{5}{2}\sqrt{13}$ or $x=-7.5+2.5\sqrt{13}$	A1	2.3
		(4)	
			(11 marks)

Notes

M1: Substitutes $x=1-\cos 2t$ and $y=\sin t\sin 2t$ into their equation from part (b) and uses $\cos 2t=\pm 2\cos^2 t\pm 1$ and $\sin 2t=2\sin t\cos t$ and $\sin^2 t=\pm 1\pm\cos^2 t$ to obtain an equation in $\cos t$ only.
Their straight line equation does not need to have the form $ax+\sqrt{2}y=b$ but must be equivalent e.g. $px+qy=r$ with $p,\ q,\ r$ non-zero.
Allow the use of $ax+\sqrt{2}y=b$ for l, i.e. with the letters a and b and allow a "made-up" a and b.

A1: Correct cubic equation in $\cos t$ with terms collected and all on one side.
Allow any multiple of this equation and the "= 0" may be implied by their attempt to solve.

M1: Uses the fact that $\left(\cos t-\frac{1}{\sqrt{2}}\right)$ oe is a factor of their cubic expression which must be of the form $\alpha\cos^3 t+\beta\cos^2 t+\gamma\cos t+\delta\quad \alpha,\beta,\gamma,\delta\neq 0$ using inspection or long division in an attempt to establish the 3 term quadratic factor **and** then solves the resulting 3TQ by a

correct method including a calculator to obtain an exact value for $\cos t$ **and** then uses $x = 1 - \cos 2t$ and $\cos 2t = \pm 2\cos^2 t \pm 1$ to obtain an exact value for x.
They must have numeric values for a and b (for l) for this mark and allow if they are "made-up" values.
Condone attempts by e.g. long division that include a remainder when dividing by $\left(\cos t - \frac{1}{\sqrt{2}}\right)$ and you do not need to check the long division/inspection as long as a 3 term quadratic factor is obtained but do not allow methods that are clearly not using the factor of $\left(\cos t - \frac{1}{\sqrt{2}}\right)$ e.g. by dividing their cubic by $\cos t$.
Do **not** allow attempts to solve their cubic on a calculator e.g.
$2\sqrt{2}\cos^3 t + 8\cos^2 t - 2\sqrt{2}\cos t - 3 = 0 \Rightarrow \cos t = \ldots \Rightarrow x = \ldots$

A1: Correct exact value and no others but ignore x = "1".