Question	Scheme	Marks	AOs
13(a)	$y^2 \rightarrow \ldots y\frac{dy}{dx}$	M1	1.1b
	$xy \rightarrow \ldots x\frac{dy}{dx} + \ldots y$	M1	1.1b
	$2x - 4y\frac{dy}{dx} - x\frac{dy}{dx} - y - 1 + 5\frac{dy}{dx} = 0$ oe e.g. $2x - 4y\frac{dy}{dx} - \left(x\frac{dy}{dx} + y\right) - 1 + 5\frac{dy}{dx} = 0$	A1	1.1b
	$(5 - 4y - x)\frac{dy}{dx} = 1 + y - 2x \Rightarrow \frac{dy}{dx} = \ldots$	M1	2.1
	$\left(\frac{dy}{dx} =\right)\frac{1 + y - 2x}{5 - 4y - x}$ oe e.g. $\frac{2x - 1 - y}{4y + x - 5}$	A1	1.1b
		(5)	

Mark (a) and (b) together.

Some candidates have a spurious "$\frac{dy}{dx} =$" appearing as their intention to differentiate e.g. $\frac{dy}{dx} = 2x - 4y\frac{dy}{dx} - x\frac{dy}{dx} - y - 1 + 5\frac{dy}{dx} = 0$. This can be condoned for the first 3 marks.

Allow equivalent notation for $\frac{dy}{dx}$ e.g. y'

M1: Differentiates y^2 to obtain $ky\frac{dy}{dx},\ k \neq 0$

M1: Differentiates xy to achieve $\alpha x\frac{dy}{dx} + \beta y,\ \ \alpha, \beta \neq 0$

A1: Fully correct differentiated equation. The "= 0" may be implied by their attempt to make $\frac{dy}{dx}$ the subject. If they rearrange before differentiating then both sides must be fully correct.

M1: For a valid attempt to make $\frac{dy}{dx}$ the subject, with **exactly 2 or 3** different terms in $\frac{dy}{dx}$

Note that here, **different** terms means terms such as $y\frac{dy}{dx}$, $x\frac{dy}{dx}$ and $5\frac{dy}{dx}$ and not e.g. $3\frac{dy}{dx}$ and $-8\frac{dy}{dx}$.

Look for $(\ldots \pm \ldots \pm \ldots)\frac{dy}{dx} = \ldots \Rightarrow \frac{dy}{dx} = \ldots$ **or** $(\ldots \pm \ldots)\frac{dy}{dx} = \ldots \Rightarrow \frac{dy}{dx} = \ldots$ which may be implied by their working.

Condone slips provided the intention is clear.

For those candidates who have a spurious $\frac{dy}{dx} = \ldots$ at the start, if they incorporate this in their rearrangement this scores M0.

A1: Correct expression in any form. $\frac{dy}{dx} =$ is not required.

Attempts to use partial differentiation in (a):

$$\mathrm{f}(x,y)=x^2-2y^2-xy-x+5y+79$$

$$\frac{\partial \mathrm{f}}{\partial x}=2x-y-1 \qquad \frac{\partial \mathrm{f}}{\partial y}=-4y-x+5$$

$$\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{\partial \mathrm{f}}{\partial x}\div\frac{\partial \mathrm{f}}{\partial y}=-\frac{2x-y-1}{-4y-x+5}$$

Score as follows:

M1: Correct structure for either partial derivative:

$$\frac{\partial \mathrm{f}}{\partial x}=\ldots x+\ldots y+\ldots \quad \text{or} \quad \frac{\partial \mathrm{f}}{\partial y}=\ldots y+\ldots x+\ldots$$

Where "…" are non-zero constants

M1: Correct structure for both partial derivatives as above.

A1: Both correct.

M1: Attempts $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{\partial \mathrm{f}}{\partial x}\div\frac{\partial \mathrm{f}}{\partial y}$

A1: Correct expression in any form.

13(b)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1+10-\frac{3\sqrt{2}}{2}-2\left(1+6\sqrt{2}\right)}{5-4\left(10-\frac{3\sqrt{2}}{2}\right)-\left(1+6\sqrt{2}\right)}$ $-\frac{\mathrm{d}x}{\mathrm{d}y}\left(\text{or } -\frac{1}{\frac{\mathrm{d}y}{\mathrm{d}x}}\right)=-\frac{5-4\left(10-\frac{3\sqrt{2}}{2}\right)-\left(1+6\sqrt{2}\right)}{1+10-\frac{3\sqrt{2}}{2}-2\left(1+6\sqrt{2}\right)}$	M1	3.1a
	$-\frac{5-4\left(10-\frac{3\sqrt{2}}{2}\right)-\left(1+6\sqrt{2}\right)}{1+10-\frac{3\sqrt{2}}{2}-2\left(1+6\sqrt{2}\right)}=-\frac{5-40+6\sqrt{2}-1-6\sqrt{2}}{11-\frac{3\sqrt{2}}{2}-2-12\sqrt{2}}$ $=\frac{36}{9-\frac{27\sqrt{2}}{2}}=\frac{36}{9-\frac{27\sqrt{2}}{2}}\times\frac{9+\frac{27\sqrt{2}}{2}}{9+\frac{27\sqrt{2}}{2}}$ or e.g. $=\frac{36}{9-\frac{27\sqrt{2}}{2}}=\frac{8}{2-3\sqrt{2}}=\frac{8}{2-3\sqrt{2}}\times\frac{2+3\sqrt{2}}{2+3\sqrt{2}}$	M1	1.1b
	$=\frac{8\left(2+3\sqrt{2}\right)}{4-18}=-\frac{8}{7}-\frac{12}{7}\sqrt{2}$	A1	2.1
		(3)	
			(8 marks)

Notes

M1: For an attempt to find the gradient of the normal.
This requires in **either order**:

- substituting the given coordinates into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ which must be in terms of x and y
- finds the negative reciprocal of the resulting expression **correctly** and award when first seen and then apply isw

Simplification is not required at this stage but may occur e.g.

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1+10-\frac{3\sqrt{2}}{2}-2\left(1+6\sqrt{2}\right)}{5-4\left(10-\frac{3\sqrt{2}}{2}\right)-\left(1+6\sqrt{2}\right)}=-\frac{1}{4}+\frac{3}{8}\sqrt{2}\Rightarrow-\frac{\mathrm{d}x}{\mathrm{d}y}=-\frac{1}{-\frac{1}{4}+\frac{3}{8}\sqrt{2}}$$

If all you see is an **incorrect** attempt at the negative reciprocal e.g. $\frac{-2+3\sqrt{2}}{8}\to\frac{8}{2+3\sqrt{2}}$

or e.g. $\frac{18-27\sqrt{2}}{-72}\to\frac{72}{27\sqrt{2}-18}$ or e.g. $-\frac{1}{4}+\frac{3}{8}\sqrt{2}\to4-\frac{8}{3\sqrt{2}}$ then score M0

This mark may be implied by sight of the correct normal gradient.
If the positioning of the minus sign means it is not perfectly clear that they have a correct negative reciprocal you may need to use your own judgement and mark positively.
You can condone work that uses a calculator for <u>this</u> mark as this mark is testing them finding the negative reciprocal however they do it.

M1: **Note that they must be working with a "changed" $\frac{\mathrm{d}y}{\mathrm{d}x}$ to access this mark and it must be of the form $\frac{\ldots}{p+q\sqrt{2}}$** where "…" can be any non-zero value or numerical expression and shows the intention to multiply both numerator and denominator by the conjugate of the denominator e.g. $\frac{\ldots}{p+q\sqrt{2}}=\frac{\ldots}{p+q\sqrt{2}}\times\frac{p-q\sqrt{2}}{p-q\sqrt{2}}$

This may be implied by e.g. $\frac{\ldots}{p+q\sqrt{2}}=\frac{\ldots\times\left(p-q\sqrt{2}\right)}{p^2-2q^2}$

Note that "changed" could be e.g. an attempt to find the reciprocal or negative reciprocal or just the negative no matter how poor.

A1: For $-\frac{8}{7}-\frac{12}{7}\sqrt{2}$ or $-\frac{12}{7}\sqrt{2}-\frac{8}{7}$ following at least one intermediate step e.g.

$$\frac{8}{2-3\sqrt{2}}\times\frac{2+3\sqrt{2}}{2+3\sqrt{2}}=\frac{8\left(2+3\sqrt{2}\right)}{-14}\text{ or e.g. }\frac{36}{9-\frac{27\sqrt{2}}{2}}\times\frac{9+\frac{27\sqrt{2}}{2}}{9+\frac{27\sqrt{2}}{2}}=\frac{36\left(9+\frac{27\sqrt{2}}{2}\right)}{81-\frac{729}{2}}$$

Do not allow a final answer of e.g. $-\frac{1}{7}\left(8+12\sqrt{2}\right)$ or e.g. $-\frac{8+12\sqrt{2}}{7}$ unless the correct form is seen first and then apply isw.