Question	Scheme	Marks	AOs
14(a)	$y=6^{-3x} \Rightarrow \left(\dfrac{dy}{dx}=\right)6^{-3x}\times -3\ln 6$	M1 A1	1.1b 1.1b
		(2)	

Note that the usual convention for logarithmic notation should be applied in this question and is as follows:

$\ln x$ or $\log_e x$: natural logs

$\log_{10} x$ or $\log x$: logs base 10

$\log_a x$: logs base a

Note that throughout this question $-3\ln 6$ may appear correctly as e.g.

$-\ln 6^3$ or $\ln 6^{-3}$ or $-\ln 216$ or $\ln\dfrac{1}{216}$

M1: Differentiates to obtain $...\ln 6\times 6^{-3x}$ **oe** e.g. $...\log_e 6\times 6^{-3x}$ where "…" is a constant and could be 1. Note that this is "or equivalent" so you may need to check carefully to see if the form is correct e.g. $...6^x\left(6^x\right)^{-4}\ln 6$ is an unusual but correct form.

Note that $\left(\dfrac{dy}{dx}=\right)...\ln 6\times 6^{-3x-1}$, although of the correct form, scores M0 in (a) and (b) unless they restart.

Note this may be obtained implicitly e.g.

$$y=6^{-3x}\Rightarrow \ln y=-3x\ln 6\Rightarrow \frac{1}{y}\frac{dy}{dx}=-3\ln 6\Rightarrow \frac{dy}{dx}=-3y\ln 6=-3\times 6^{-3x}\ln 6$$

or equivalent with $\log_e 6$ as $\ln 6$

The M1 is scored once they have the correct form in terms of x only (not x and y)

Note this may also be obtained using "e":

$$y=6^{-3x}=e^{\ln 6^{-3x}}=e^{-3x\ln 6}\Rightarrow \frac{dy}{dx}=e^{-3x\ln 6}\times -3\ln 6$$

Note this may also be obtained by making x the subject:

$$y=6^{-3x}\Rightarrow \log_6 y=-3x\Rightarrow x=-\frac{1}{3}\frac{\ln y}{\ln 6}\Rightarrow \frac{dx}{dy}=-\frac{1}{3y\ln 6}\Rightarrow \frac{dy}{dx}=-3\times 6^{-3x}\ln 6$$

Do **not** allow $y=6^{-3x}\Rightarrow\left(\dfrac{dy}{dx}=\right)6^{-3x}\times -3\log 6$ unless the $\log 6$ subsequently becomes $\ln 6$ or $\log_e 6$

A1: Correct expression. The "$\dfrac{dy}{dx}=$" is not required. Allow any correct expressions e.g.

$\left(6^{-3}\right)^x\times \ln 6^{-3}$ or $e^{-3x\ln 6}\times -3\ln 6$

Award this mark as soon as a correct expression is seen and apply isw.

Note that they may use a substitution e.g.

$$u=-3x\Rightarrow y=6^{-3x}=6^u\Rightarrow\left(\frac{dy}{du}=\right)6^u\times\ln 6\Rightarrow \frac{dy}{dx}=6^{-3x}\times\ln 6\times\frac{du}{dx}=-3\times 6^{-3x}\ln 6$$

Or e.g. $u=-3x\Rightarrow y=6^{-3x}=6^u\Rightarrow\left(\dfrac{dy}{dx}=\right)6^u\times\ln 6\times\dfrac{du}{dx}\Rightarrow\dfrac{dy}{dx}=-3\times 6^{-3x}\ln 6$

In either case, award the M mark for reaching $\ldots\ln 6\times 6^{-3x}$ oe e.g. $\ldots\log_e 6\times 6^{-3x}$ as in the main scheme and then A1 if correct in terms of x.

14(b)	$6^{-3x}\times -3\ln 6=-1\Rightarrow 6^{-3x}=\dfrac{1}{3\ln 6}\Rightarrow -3x\ln 6=\ln\left(\dfrac{1}{3\ln 6}\right)$ or e.g. $6^{-3x}\times -3\ln 6=-1\Rightarrow 6^{-3x}=\dfrac{1}{3\ln 6}\Rightarrow -3x=\log_6\left(\dfrac{1}{3\ln 6}\right)$ or e.g. $6^{-3x}\times -3\ln 6=-1\Rightarrow 6^{-3x}=\dfrac{1}{3\ln 6}\Rightarrow \log\left(6^{-3x}\right)=\log\left(\dfrac{1}{3\ln 6}\right)$ $\Rightarrow -3x\log 6=\log\left(\dfrac{1}{3\ln 6}\right)$	M1	3.1a
	$\Rightarrow x=-\dfrac{1}{3\ln 6}\ln\left(\dfrac{1}{3\ln 6}\right)$ or e.g. $-\dfrac{1}{3}\log_6\left(\dfrac{1}{3\ln 6}\right)$ or e.g. $\dfrac{\log\left(\frac{1}{3\ln 6}\right)}{-3\log 6}$	A1	2.1
		(2)	
			(4 marks)

Note that for both marks in (b), brackets are not required around the arguments of the logarithms as long as they are single terms e.g. $\ln\left(\dfrac{1}{3\ln 6}\right)$ **and** $\ln\dfrac{1}{3\ln 6}$ **are both acceptable.**

Do not condone "misreads" of –1 as +1 in this part.

M1: Correct strategy to obtain an equation where x is no longer an index.

It must come from an equation of the form $-k\ln p\left(6^{-3x}\right)=-1$ or $k\times\ln p\left(6^{-3x}\right)=1$

where k and p are constants with $k>0$, $k\neq 1$ and $p>1$ **or equivalent**

e.g. $-k\ln p\mathrm{e}^{-3x\ln 6}=-1$ is equivalent.

e.g. takes ln's or logs base 6 or takes logs of **both sides** using any other base so that x is not an index.

Look for e.g. $-k\ln p\times 6^{-3x}=-1\to 6^{-3x}=\dfrac{1}{k\ln p}\to \ln 6^{-3x}=\ln\dfrac{1}{k\ln p}\to -3x\ln 6=\ln\dfrac{1}{k\ln p}$

Or e.g. $-k\ln p\times 6^{-3x}=-1\to 6^{-3x}=\dfrac{1}{k\ln p}\to \log_6 6^{-3x}=\log_6\dfrac{1}{k\ln p}\to -3x=\log_6\dfrac{1}{k\ln p}$

$$\left(\text{or } -3x\log_6 6=\log_6\dfrac{1}{k\ln p}\right)$$

Or e.g. $-k\ln p6^{-3x}=-1\to 6^{-3x}=\dfrac{1}{k\ln p}\to \log 6^{-3x}=\log\dfrac{1}{k\ln p}\to -3x\log 6=\log\dfrac{1}{k\ln p}$

Condone correct work that is non-exact for this mark e.g. using ln 6 as 1.791…

Do **not** condone work that involves taking logarithms of **negative** numbers

e.g. $6^{-3x}\times -3\ln 6=-1\Rightarrow -6^{-3x}=-\dfrac{1}{3\ln 6}\Rightarrow -3x\ln(-6)=\ln\left(-\dfrac{1}{3\ln 6}\right)$

Note that e.g. $\ln\left(-\dfrac{1}{\ln\dfrac{1}{216}}\right)$ is ok as $-\dfrac{1}{\ln\dfrac{1}{216}}>0$

Beware $\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}=\right)k\ln p\times 6^{-3x-1}$ although of the correct form, is not acceptable for this mark.

A1: Correct **exact** value in any form.
Award this mark as soon as a correct value is seen and apply isw.
Note that there are many correct forms so you need to look carefully at their answer:
e.g. these are also correct: $x=\dfrac{1}{3\ln 6}\ln(3\ln 6)$, $x=-\dfrac{1}{3}\log_6\dfrac{1}{\ln 216}$, $x=\dfrac{1}{3}\log_6 \ln 216$
Note that $\ln 6$ may appear as e.g. $\ln 3+\ln 2$
Note that for reference $x = 0.3128787463\ldots$ and **could** be a way of checking if their exact expression is correct.

In part (b), condone slips in rearranging as long as the key step in using logarithms to reach the point where *x* is no longer an index is correct.
e.g. we would allow M1A0 in (b) for:

$$\textbf{(a)}\ \ y=6^{-3x}=\left(6^{-3}\right)^x=\left(\frac{1}{216}\right)^x\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\ln\left(\frac{1}{216}\right)\times\left(\frac{1}{216}\right)^x\ \ \textbf{(M1A1)}$$

$$\textbf{(b)}\ \ -\ln(216)\times(216)^{-x}=-1\Rightarrow(216)^{-x}=\ln 216-1\Rightarrow x=-\frac{\ln(\ln 216-1)}{\ln 216}\ \ \textbf{(M1A0)}$$

<u>Some common incorrect derivatives that score no marks in (a) or (b)</u>

$$y=6^{-3x}\Rightarrow\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)-3x6^{-3x-1}$$

$$y=6^{-3x}\Rightarrow\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)-3\times\ln 6\left(6^{-3x-1}\right)$$

$$y=6^{-3x}\Rightarrow\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)-3x6^{-3x}$$

$$y=6^{-3x}\Rightarrow\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)-3\times 6^{-3x}$$

<u>Note that there may be more unusual valid methods e.g. this has been seen:</u>

$$\textbf{(a)}\ \ y=6^{-3x}=\left(\frac{1}{216}\right)^x=\mathrm{e}^{x\ln\frac{1}{216}}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\ln\frac{1}{216}\mathrm{e}^{x\ln\frac{1}{216}}$$

Although disguised, the form of the derivative is correct i.e. $\ln\dfrac{1}{216}\mathrm{e}^{x\ln\frac{1}{216}}\equiv-3\ln 6\times 6^{-3x}$

$$\textbf{(b)}\ \ \frac{\mathrm{d}y}{\mathrm{d}x}=\ln\frac{1}{216}\mathrm{e}^{x\ln\frac{1}{216}}=-1\Rightarrow-\ln 216\mathrm{e}^{x\ln\frac{1}{216}}=-1\Rightarrow\ln 216\mathrm{e}^{x\ln\frac{1}{216}}=1$$

$$e^{x\ln\frac{1}{216}} = \frac{1}{\ln 216} \Rightarrow \left(\frac{1}{216}\right)^x = \frac{1}{\ln 216} \Rightarrow x\ln\left(\frac{1}{216}\right) = \ln\left(\frac{1}{\ln 216}\right) \Rightarrow x = \frac{\ln\left(\frac{1}{\ln 216}\right)}{\ln\left(\frac{1}{216}\right)}$$

In this part they start with an equation of the required form and use a correct strategy to make *x* a coefficient so the M1 is scored and the answer is correct A1

Remember you can always check their answer in (b) by comparing it with 0.3128787463… **if necessary.**

<u>Some commonly seen responses with marks:</u>

$y = 6^{-3x} \Rightarrow \left(\frac{dy}{dx} = \right) 6^{-3x} \times -3\ln 6 = 18^{-3x}\ln 6$ scores M1A1 isw in (a) but

$y = 6^{-3x} \Rightarrow \left(\frac{dy}{dx} = \right) 18^{-3x}\ln 6$ scores M0A0 in (a)

If $\frac{dy}{dx} = 18^{-3x}\ln 6$ is used in (b) this scores M0A0

$y = 6^{-3x} \Rightarrow \left(\frac{dy}{dx} = \right) 6^{-3x} \times \ln 6$ scores M1A0 in (a) and then M0A0 in (b) (if no restart)