Question	Scheme	Marks	AOs
15(a)	$x^2+y^2+6x-4y-62=0 \Rightarrow (x+3)^2+(y-2)^2=...$	M1	1.1b
(i)	Centre is (−3, 2)	A1	1.1b
(ii)	$r=5\sqrt{3}$ oe e.g. $r=\sqrt{75}$	B1	1.1b
		(3)	

Notes

Mark parts (a)(i) and (a)(ii) together.

Note this is being marked M1A1B1 not M1A1A1

(a)(i)

M1: Attempts to complete the square on **both** x and y terms.
Accept $(x\pm3)^2+(y\pm2)^2$ or imply this mark for a centre of $(\pm3,\pm2)$
Condone $(x\pm3)^2....(y\pm2)^2$ where "..." could be , or even −

A1: Correct centre $(-3,\ 2)$. Condone missing brackets e.g. −3, 2 or unusual notation e.g. $(-3;\ 2)$ or as a column vector e.g. $\begin{pmatrix}-3\\2\end{pmatrix}$ but do **not** allow coordinates the wrong way round.
Allow e.g. $x=-3,\ y=2$

(a)(ii)

B1: $\sqrt{75}$ or exact equivalent e.g. $5\sqrt{3}$. Isw once the correct answer is seen e.g. if they subsequently change to decimals. But do not isw if they subsequently e.g. double or halve their answer.
Do not allow $\pm\sqrt{75}$

Question	Scheme	Marks	AOs
15(b)	$y=2x+k \Rightarrow x^2+(2x+k)^2+6x-4(2x+k)-62=0$ or e.g. $y=2x+k \Rightarrow (x+3)^2+(2x+k-2)^2=75$ or e.g. $y=2x+k \Rightarrow \left(\frac{y-k}{2}\right)^2+y^2+6\left(\frac{y-k}{2}\right)-4y-62=0$ or e.g. $y=2x+k \Rightarrow \left(\frac{y-k}{2}+3\right)^2+(y-2)^2=75$	M1	3.1a
	$5x^2+(4k-2)x+k^2-4k-62=0$ or $5y^2-(2k+4)y+k^2-12k-248=0$	A1	1.1b
	$b^2-4ac=0 \Rightarrow (4k-2)^2-4\times5\times(k^2-4k-62)=0$ $\Rightarrow k^2-16k-311=0 \Rightarrow k=8\pm5\sqrt{15}$	dM1 A1	3.1a 2.2a

	or $$b^2-4ac=0 \Rightarrow (2k+4)^2-4\times5\times(k^2-12k-248)=0$$ $$\Rightarrow k^2-16k-311=0 \Rightarrow k=8\pm5\sqrt{15}$$		
	$$\{k: 8-5\sqrt{15}<k\}\cap\{k: k<8+5\sqrt{15}\}$$ or e.g. $$\{k: 8-5\sqrt{15}<k<8+5\sqrt{15}\}$$	A1	2.5
		(5)	
			(8 marks)

Notes

M1: Solves simultaneously to obtain an equation in one variable.
See scheme for some examples e.g.

- substitutes $y = 2x + k$ into the given circle equation or into their factorised form.
- makes x the subject of $y = 2x + k$ and substitutes x in terms of y and k into the given circle equation or into their factorised form

Condone miscopies of the circle equation and/or $y = 2x + k$ as long as the intention is clear and the resulting equation is in terms of x and k **or** y and k.

A1: Correct simplified 3TQ with terms collected but not necessarily all on one side.
Note that the x or y terms do not need the coefficient factorised so that e.g. $4kx-2x$ is acceptable for $(4k-2)x$. The x terms need to be on the same side as each other and the constant terms also need to be on the same side as each other.
Some examples of acceptable equations are

- $4kx+5x^2-2x=62+4k-k^2$
- $k^2-12k-248=(2k+4)y-5y^2$
- $5x^2-62-4k+k^2=2x-4kx$

A correct equation may be implied by a correct discriminant or equivalent.
e.g. $5x^2+4kx-8x+6x=62+4k-k^2$ would score A0 unless followed by a correct discriminant or equivalent.
Condone the omission of "= 0" if the correct equation is implied by a correct attempt at the discriminant or equivalent.

dM1: Uses b^2-4ac where

- a is a constant with no k's
- b is of the form $pk+q,\ p,q\neq0$
- c is of the form $\alpha k^2+\beta k+\gamma,\ \alpha,\beta,\gamma\neq0$

AND finds 2 critical values from a 3TQ in k.
May be seen in an attempt at e.g. $b^2-4ac...0$ where "…" is any inequality or "=" or equivalent e.g. $b^2...4ac$
Allow any acceptable method for solving their 3TQ including a calculator.
This may be implied by their critical values so you may need to check if their 3TQ is incorrect. Condone inexact answers for this mark – allow 2sf accuracy.
Depends on the first method mark.

A1: Correct **exact** values for k. Must be simplified as shown but allow exact equivalents e.g. $8\pm\sqrt{375}$ but condone use of x here e.g. $x=8\pm\sqrt{375}$ for this mark.

A1: Correct answer with **exact values** using <u>set notation in terms of k</u>.
Allow $8\pm\sqrt{375}$ for $8\pm5\sqrt{15}$

Allow as shown in the main scheme but also allow equivalent set notation e.g.
$\{k : k \in \mathbb{R}, 8-5\sqrt{15} < k < 8+5\sqrt{15}\}$, $\quad k \in \left(8-\sqrt{375}, 8+\sqrt{375}\right)$
and allow "|" for ":" and allow the "k:" or "$k \in$" to be missing
e.g. $\{8-5\sqrt{15} < k < 8+5\sqrt{15}\}$ and $\left(8-5\sqrt{15}, 8+5\sqrt{15}\right)$ are both acceptable.
But $\{k : k < 8+5\sqrt{15}\} \cup \{k : k > 8-5\sqrt{15}\}$ or $\{k :\ 8-5\sqrt{15}, 8+5\sqrt{15}\}$ score A0
Do **not** allow solutions not in set notation such as $8-5\sqrt{15} < k < 8+5\sqrt{15}$
Do **not** allow use of non-strict inequalities.

Note that it is possible to find the critical values by alternative methods:

Alternative 1 using implicit differentiation and the radial line.

	$x^2+y^2+6x-4y-62=0 \Rightarrow 2x+2y\dfrac{\mathrm{d}y}{\mathrm{d}x}+6-4\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$	M1	3.1a
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=2 \Rightarrow x+2y-1=0$	A1	1.1b
	$x+2y-1=0,\ x^2+y^2+6x-4y-62=0$ $\Rightarrow 5x^2+30x-255=0$ or $5y^2-20y-55=0$ $5x^2+30x-255=0 \Rightarrow x=-3\pm 2\sqrt{15} \Rightarrow y=2\mp\sqrt{15}$ $k=y-2x \Rightarrow k=2\mp\sqrt{15}-2\left(-3\pm 2\sqrt{15}\right)=8\mp 5\sqrt{15}$ or $5y^2-20y-55=0 \Rightarrow y=2\pm\sqrt{15} \Rightarrow x=-3\mp 2\sqrt{15}$ $k=y-2x \Rightarrow k=2\pm\sqrt{15}-2\left(-3\mp 2\sqrt{15}\right)=8\pm 5\sqrt{15}$	dM1 A1	3.1a 2.2a
	$\{k : 8-5\sqrt{15} < k\} \cap \{k : k < 8+5\sqrt{15}\}$ or e.g. $\{k : 8-5\sqrt{15} < k < 8+5\sqrt{15}\}$	A1	2.5

Notes

M1: Differentiates implicitly with at least $y^2 \to \ldots y\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and $-4y \to \ldots\dfrac{\mathrm{d}y}{\mathrm{d}x}$

A1: Substitutes $\dfrac{\mathrm{d}y}{\mathrm{d}x}=2$ to obtain a correct equation for the radial line.

dM1: A complete method to find the critical values e.g.

- solves their radial line equation simultaneously with the given circle equation or their factorised form to obtain a 3TQ in x or y
- attempts to solve the resulting 3TQ in x or y by any acceptable method including a calculator where the "= 0" in their 3TQ may be implied by their attempt to solve
- substitutes their x or y back into their $x+2y-1=0$ to obtain the value of the other variable to obtain 2 sets of values for x **and** y
- uses $y = 2x + k$ (must be this equation) with both of their pairs of values to obtain the 2 critical values for k

Condone working with inexact values for this mark.

	Depends on the first method mark. **A1**: As main scheme. **A1**: As main scheme.		
	Alternative 2 implicit differentiation and radial line.		
	$x^2+y^2+6x-4y-62=0 \Rightarrow 2x+2y\frac{dy}{dx}+6-4\frac{dy}{dx}=0$	M1	3.1a
	$\frac{dy}{dx}=2 \Rightarrow x+2y-1=0$	A1	1.1b
	$x+2y-1=0,\ y=2x+k \Rightarrow x=\frac{1-2k}{5},\ y=\frac{k+2}{5}$ $\Rightarrow \left(\frac{1-2k}{5}\right)^2+\left(\frac{k+2}{5}\right)^2+6\left(\frac{1-2k}{5}\right)-4\left(\frac{k+2}{5}\right)-62=0$ $\Rightarrow k^2-16k-311=0 \Rightarrow k=8\pm5\sqrt{15}$	dM1 A1	3.1a 2.2a
	$\{k:8-5\sqrt{15}<k\}\cap\{k:k<8+5\sqrt{15}\}$ or e.g. $\{k:8-5\sqrt{15}<k<8+5\sqrt{15}\}$	A1	2.5

Notes

M1: Differentiates implicitly with at least $y^2 \to ...y\frac{dy}{dx}$ and $-4y \to ...\frac{dy}{dx}$

A1: Substitutes $\frac{dy}{dx}=2$ to obtain a correct equation for the radial line.

dM1: A complete method to find the critical values e.g.

- solves their radial line simultaneously with the equation of *l* to obtain *x* and *y* in terms of *k* and substitutes these into the given circle equation or their factorised form to obtain a 3TQ in *k*
- attempts to solve the resulting 3TQ in *k* by any acceptable method including a calculator where the "= 0" in their 3TQ may be implied by their attempt to solve to obtain the 2 critical values for *k*

Condone working with inexact values for this mark.
Depends on the first method mark.

A1: As main scheme.
A1: As main scheme.

	Alternative 3 using equation of radial line without implicit differentiation.		
	$y=2x+k \Rightarrow m_N=-\frac{1}{2} \Rightarrow y-2=-\frac{1}{2}(x+3)$	M1 A1	3.1a 1.1b

	$(x+3)^2+(y-2)^2=75 \Rightarrow (x+3)^2+\frac{1}{4}(x+3)^2=75 \Rightarrow (x+3)^2=60$ $\Rightarrow x=-3\pm 2\sqrt{15} \Rightarrow y=2\mp\sqrt{15}$ $k=y-2x \Rightarrow k=2\mp\sqrt{15}-2\left(-3\pm 2\sqrt{15}\right)=8\pm 5\sqrt{15}$ or $(x+3)^2+(y-2)^2=75 \Rightarrow 4(y-2)^2+(y-2)^2=75 \Rightarrow (y-2)^2=15$ $\Rightarrow y=2\pm\sqrt{15} \Rightarrow x=-3\mp 2\sqrt{15}$ $k=y-2x \Rightarrow k=2\pm\sqrt{15}-2\left(-3\mp 2\sqrt{15}\right)=8\pm 5\sqrt{15}$	dM1 A1	3.1a 2.2a
	$\{k:8-5\sqrt{15}<k\}\cap\{k:k<8+5\sqrt{15}\}$ or e.g. $\{k:8-5\sqrt{15}<k<8+5\sqrt{15}\}$	A1	2.5

Notes

M1: Uses a gradient of $-\frac{1}{2}$ with the coordinates of the centre of C to form the equation of the radial line. Follow through their centre.
If using $y = mx + c$ must proceed as far as $c = \ldots$

A1: Correct equation in any form.

dM1: A complete method to find the critical values e.g.

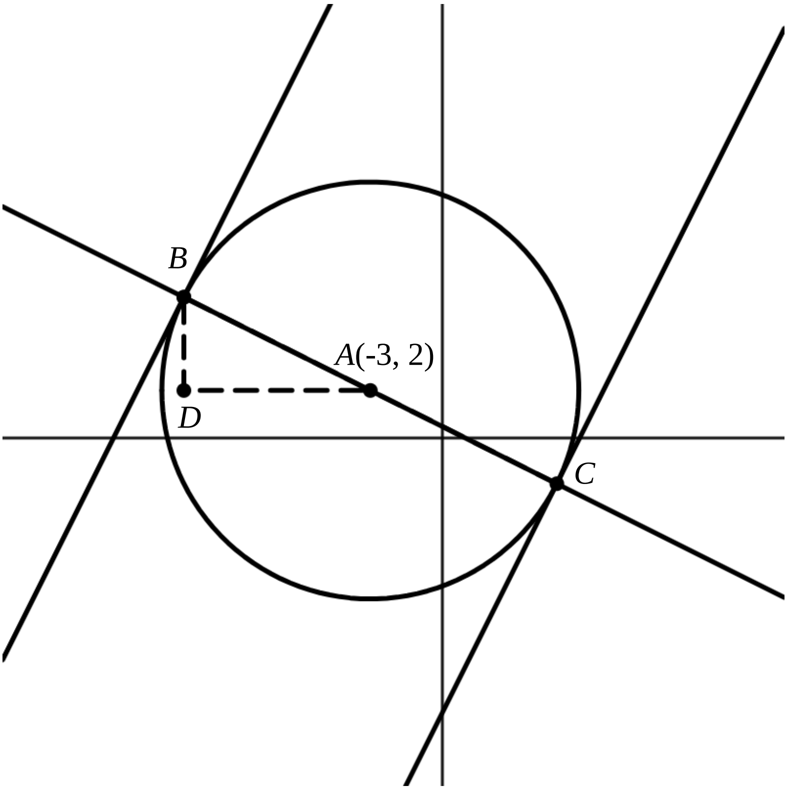
- solves their radial line simultaneously with the given circle equation or their factorised form to obtain a 3TQ in x or y
- attempts to solve the resulting 3TQ in x or y by any acceptable method including a calculator where the "= 0" in their 3TQ may be implied by their attempt to solve
- substitutes their x or y back into their $x+2y-1=0$ to obtain the value of the other variable to obtain 2 sets of values for x **and** y
- uses $y = 2x + k$ (must be this equation) with both of their pairs of values to obtain the 2 critical values for k

Condone working with inexact values for this mark.
Depends on the first method mark.

A1: As main scheme.

A1: As main scheme.

Alternative 4 using intersections of radii and tangential lines.



	Gradient BC is $-\frac{1}{2} \to BD = \alpha,\ AD = 2\alpha$ $\alpha^2 + (2\alpha)^2 = "\sqrt{75}"^2 \Rightarrow \alpha = \sqrt{15}$ So B is $\left(-3-2\sqrt{15}, 2+\sqrt{15}\right)$ and C is $\left(-3+2\sqrt{15}, 2-\sqrt{15}\right)$	M1 A1	3.1a 1.1b
	$k = y - 2x \Rightarrow k = 2 \pm \sqrt{15} - 2\left(-3 \mp 2\sqrt{15}\right) = 8 \pm 5\sqrt{15}$	dM1 A1	3.1a 2.2a
	$\left\{k : 8 - 5\sqrt{15} < k\right\} \cap \left\{k : k < 8 + 5\sqrt{15}\right\}$ or e.g. $\left\{k : 8 - 5\sqrt{15} < k < 8 + 5\sqrt{15}\right\}$	A1	2.5

Notes

M1: Uses a gradient of $-\frac{1}{2}$ to deduce that AD:BD = 2:1 and uses their radius with Pythagoras to find the vertical and horizontal distances between A and B and then finds the coordinates of B and C using their centre and their $"\alpha"$ using $x = "-3" \mp 2\alpha$ and $y = "2" \pm \alpha$

A1: Correct coordinates of B and C.

dM1: Uses $y = 2x + k$ (must be this equation) with both pairs of coordinates to find the 2 critical values of k.
Condone inexact answers for this mark.
Depends on the first method mark.

A1:	As main scheme.
A1:	As main scheme.

15b Diagram for reference: