

6. The discrete random variable X has the following probability distribution

x	a	b	c
$P(X=x)$	$\log_{36} a$	$\log_{36} b$	$\log_{36} c$

where

- a, b and c are distinct integers ($a < b < c$)
- all the probabilities are greater than zero

(a) Find

- the value of a
- the value of b
- the value of c

Show your working clearly.

(5)

The independent random variables X_1 and X_2 each have the same distribution as X

(b) Find $P(X_1 = X_2)$

(2)

$$\text{(a)} \quad \log_{36} a + \log_{36} b + \log_{36} c = 1 \quad (1 \text{ mark})$$

$$\log_{36} (abc) = 1 \Rightarrow abc = 36 \quad (1 \text{ mark})$$

Factors of 36 are ~~1~~ 2, 3, 4, 6, 9, 12, 18

Given all probabilities > 0
 $\log(1) = 0$, so $a, b, c \neq 1$ (1 mark)

Only 3 distinct integers that multiply to give 36 are 2, 3, 6

so (i) $a = 2$

(a)(ii) $b = 3$

(a)(iii) $c = 6$

(2 marks)

$$\text{(b)} \quad P(X_1 = X_2) = P(a, a) + P(b, b) + P(c, c)$$

$$= P(a)^2 + P(b)^2 + P(c)^2 \quad \left(\begin{array}{l} \text{because same} \\ \text{distribution \&} \\ \text{independent} \end{array} \right)$$

$$= (\log_{36} 2)^2 + (\log_{36} 3)^2 + (\log_{36} 6)^2 \quad (1 \text{ mark})$$

$$= (0.1934\dots)^2 + (0.3065\dots)^2 + (0.5)^2 = 0.3814\dots = 0.381 \text{ 3sf} \quad (1 \text{ mark})$$