

5. Kim and Tom are both learning to tune a violin.

Kim's teacher asks her to tune the *A-string* of her violin to the correct frequency in hertz (Hz).

When Kim tunes the *A-string*, its frequency may be modelled by a Normal distribution.

When Kim first starts learning, she tunes the *A-string* with a mean frequency of 443 Hz and a standard deviation of 6 Hz.

The correct frequency for the *A-string* is 440 Hz.

Find the probability that Kim tunes the *A-string*

- (a) lower than the correct frequency,

(a) Let  $F$  be Frequency *A-string* is tuned to  
 $F \sim N(443, 6^2)$   
 $P(F < 440) =$  (1 mark)  
 fx-991EX: MENU 7-Dist/NormalCD/ } = 0.30853...  
 fx-CG50: DIST/NORM/Ncd } = 0.3093sf (1 mark) (2)

- (b) more than 5 Hz away from the correct frequency.

(b)  $P(F < 435) + P(F > 445) = 1 - P(435 < F < 445)$  NormalCD or Ncd 1 - 0.53934... (2)

After practising for a month, Kim tunes the *A-string* with a standard deviation of 4.5 Hz.

She claims that the mean frequency when she tunes the *A-string* is now less than 443 Hz.

(b) ctd  
 $= 0.46065...$   
 $= 0.461$  3sf (2 marks)

Kim's teacher asks Kim to tune the *A-string* 20 times and finds that the mean frequency is 442 Hz.

(c)  $F \sim N(\mu, 4.5^2)$   $H_0: \mu = 443$   $H_1: \mu < 443$  (1 mark)

- (c) Test at the 5% level of significance whether or not there is evidence to support Kim's claim.

(c) ctd Under  $H_0$ , sample mean  $\bar{X} \sim N(443, (\frac{4.5}{\sqrt{20}})^2)$  (1 mark)

You should state your hypotheses and show your working clearly.

(c) ctd  $P(\bar{X} < 442)$  NormalCD or Ncd 0.1601... = 16.01...% (1 mark) (4)

When Tom tunes the *A-string*, its frequency,  $T$  Hz, may be modelled by  $T \sim N(\mu, \sigma^2)$

Given that  $P(T < 438) = 0.2$  and that  $P(T > 445) = 0.1$

- (d) find the value of  $\mu$  and the value of  $\sigma$ , giving your answers, in Hz, to 1 decimal place.

(c) ctd 16.01...% > 5% so is not significant at 5% level. There is insufficient evidence to support Kim's claim (1 mark)

(6)

(d)  $T \sim N(\mu, \sigma^2) \Rightarrow Z = \frac{T - \mu}{\sigma} \sim N(0, 1^2)$  ← standardised

fx-991EX: MENU 7-Dist/InverseNormal/ }  $\sigma = 1$  }  $P(Z < z) = 0.2$   
 fx-CG50: MENU Stats/DIST/NORM/InvN }  $\mu = 0$  } Area = 0.2  $\Rightarrow z = -0.8416...$

so,  $\frac{438 - \mu}{\sigma} = -0.8416...$  (2 marks)

$P(Z > z) = 0.1 \Rightarrow P(Z < z) = 0.9$  InverseNormal InvN  $z = 1.2815...$

so,  $\frac{445 - \mu}{\sigma} = 1.2815...$  (2 marks)

Solving simultaneously,

$$\begin{array}{rcl} 438 - \mu & = & -0.8416... \sigma \\ -(445 - \mu & = & 1.2815... \sigma) \\ \hline -7 & = & -2.1231... \sigma \end{array}$$

$$\sigma = \frac{-7}{-2.1231...} = 3.296... = 3.3 \text{ Hz 1dp} \quad (1 \text{ mark})$$

$$\begin{array}{l} \mu = 438 - (-0.8416... \times 3.296...) \\ \mu = 440.77... = 440.8 \text{ Hz 1dp} \quad (1 \text{ mark}) \end{array}$$