

2. A manufacturer uses a machine to make metal rods.

The length of a metal rod, L cm, is normally distributed with

- a mean of 8 cm
- a standard deviation of x cm

(a) $L \sim N(8, x^2)$

$P(L < 7.902) = 0.025$

$P(Z < \frac{7.902 - 8}{x}) = 0.025$

where $Z \sim N(0, 1^2)$

Given that the proportion of metal rods less than 7.902 cm in length is 2.5%

(a) show that $x = 0.05$ to 2 decimal places.

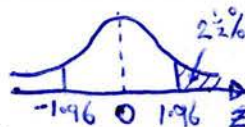
(a) contd From "Percentage Points of the Normal Distribution" in Formula Booklet

$P(Z > 1.9600) = 0.025$

(2)

(b) Calculate the proportion of metal rods that are between 7.94 cm and 8.09 cm in length.

(a) contd



(1)

The cost of producing a single metal rod is 20p

A metal rod

- where $L < 7.94$ is sold for scrap for 5p
- where $7.94 \leq L \leq 8.09$ is sold for 50p
- where $L > 8.09$ is shortened for an extra cost of 10p and then sold for 50p

By symmetry,

$P(Z < -1.96) = 0.025$ (1 mark)

$\Rightarrow \frac{7.902 - 8}{x} = -1.96$
 $\Rightarrow x = 0.05$ (1 mark)

(c) Calculate the expected profit per 500 of the metal rods.

Give your answer to the nearest pound.

(5)

The same manufacturer makes metal hinges in large batches.

The hinges each have a probability of 0.015 of having a fault.

A random sample of 200 hinges is taken from each batch and the batch is accepted if fewer than 6 hinges are faulty.

The manufacturer's aim is for 95% of batches to be accepted.

(d) Explain whether the manufacturer is likely to achieve its aim.

(4)

(b) From (a) $L \sim N(8, (0.05)^2)$

For $P(7.94 \leq L \leq 8.09)$

fx-991EX: Menu 7-Dist/NormalCD

fx-CG50: Menu 2-Stats/DIST/NormalCD

$\} = 0.8490...$

$= 0.849$ 3sf (1 mark)

(d) Let X be no. of hinges in batch of 200 with a fault

$X \sim B(200, 0.015)$ (1 mark)

$P(X < 6) = P(X \leq 5)$

because Binomial is discrete distribution

(1 mark) from BinomialCD

on Calculator

$0.9176...$

(1 mark)

$0.9176... < 0.95$ (95%)

so manufacturer is not likely to achieve its aim (1 mark)

Question 2 continued

The **cost** of producing a single metal rod is 20p

A metal rod

- where $L < 7.94$ is **sold** for scrap for 5p
- where $7.94 \leq L \leq 8.09$ is **sold** for 50p
- where $L > 8.09$ is shortened for an extra **cost** of 10p and then **sold** for 50p

(c) Calculate the expected profit per 500 of the metal rods.

Give your answer to the nearest pound.

(5)

$$(c) \quad L \sim N(8, (0.05)^2)$$

$$P(L > 8.09) = 1 - P(L \leq 8.09) \quad \begin{array}{l} \text{from Normal CD} \\ \text{on calculator} \end{array} \quad \begin{array}{l} 1 - 0.964... = 0.03593... \\ (1 \text{ mark}) \end{array}$$

$P(L > 8.09)$	$0.03593...$	$\times (50p - 20p - 10p) = 0.7186...$
$P(7.94 \leq L \leq 8.09)$	$\begin{array}{l} \text{from (b)} \\ 0.8490... \end{array}$	$\times (50p - 20p) = 25.47...$
$P(L < 7.94)$	$0.1150...$	$\times (5p - 20p) = -1.726...$

$$P(L < 7.94) \quad \begin{array}{l} \text{from Normal CD} \\ \text{on calculator} \end{array} \quad \text{OR} = 1 - 0.03593... - 0.8490... = 0.1150... \quad (1 \text{ mark})$$

Expected profit per rod

$$= 0.7186... + 25.47... - 1.726... = 24.46... p. \quad (1 \text{ mark})$$

Expected profit per 500 rods

$$= 500 \times 24.46... p. = 12231.2... p. = \pounds 122 \text{ to nearest } \pounds \quad (2 \text{ marks})$$