

4. A dentist knows from past records that 10% of customers arrive late for their appointment.

A new manager believes that there has been a change in the proportion of customers who arrive late for their appointment.

A random sample of 50 of the dentist's customers is taken.

(a) Write down

- a null hypothesis corresponding to no change in the proportion of customers who arrive late
- an alternative hypothesis corresponding to the manager's belief

(1)

(b) Using a 5% level of significance, find the critical region for a two-tailed test of the null hypothesis in (a)

You should state the probability of rejection in each tail, which should be less than 0.025

(3)

(c) Find the actual level of significance of the test based on your critical region from part (b)

(1)

The manager observes that 15 of the 50 customers arrived late for their appointment.

(d) With reference to part (b), comment on the manager's belief.

(1)

(a) Let  $L$  be no. of customers who arrive late  
 $L \sim B(50, p)$

$$H_0: p = 0.1, H_1: p \neq 0.1 \quad (1 \text{ mark})$$

(b) From "Binomial Cumulative Distribution Function" table in Formula Booklet for  $n=50, p=0.10$ , Under  $H_0$ ,

$$P(L \leq 0) = 0.0052 \quad P(L \geq 11) = 1 - P(L \leq 10) = 1 - 0.9906 = 0.0094$$

$$P(L \leq 1) = 0.0338 \quad P(L \geq 10) = 1 - P(L \leq 9) = 1 - 0.9755 = 0.0245$$

$$0.0338 > 0.025$$

$$P(L \geq 9) = 1 - P(L \leq 8) = 1 - 0.9421 = 0.0579$$

so critical region is  $L=0$   
 at lower end  
 (1 mark)

$0.0579 > 0.025$  so critical region (1 mark)  
 is  $L \geq 10$  at upper end

(c) Actual level of significance = probability of results in critical region under  $H_0$   
 $= 0.0052 + 0.0245 = 0.0297 \quad (2.97\%) \quad (1 \text{ mark})$

(d)  $15 \geq 10$ , so 15 is in the critical region  
 there is evidence to support the manager's belief  
 (1 mark)