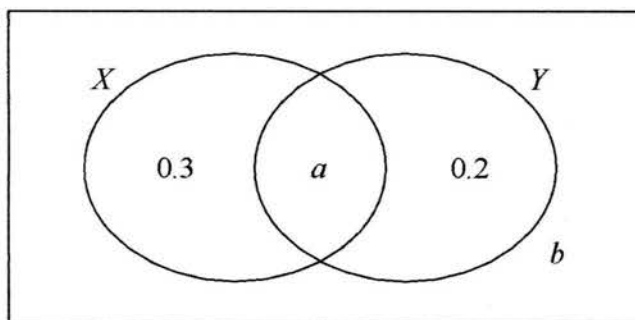


3. The Venn diagram shows probabilities related to two events, X and Y



(a) (i) State the value of $P(Y \cap X')$

(a)(i) 0.2 (1 mark)

(ii) Find $P(Y|X')$ giving your answer in terms of b

(a)(ii) $P(A|B) = \frac{P(A \cap B)}{P(B)}$ (3)

Given that $P(X|Y) = P(Y|X')$

(b) show that $ab = 0.04$

so, $P(Y|X') = \frac{P(Y \cap X')}{P(X')}$ (1 mark)

Given also that $a > b$

$= \frac{0.2}{b+0.2}$ (1 mark)

(c) find the value of a and the value of b
Show your working clearly.

(b) $P(X|Y) = \frac{P(X \cap Y)}{P(Y)}$ (5)

(d) Hence show that X and Y are not independent.

$= \frac{a}{a+0.2}$ (2)

(b) contd Given $\frac{a}{a+0.2} = \frac{0.2}{b+0.2}$ (1 mark)

$ab + 0.2a = 0.2b + 0.04$ (1 mark)

$ab = 0.04$ (1 mark)

(c) Probabilities sum to 1, so $0.3 + a + 0.2 + b = 1$
 $\Rightarrow a + b = 0.5$ (1 mark)

$a + b = 0.5 \Rightarrow a = 0.5 - b$

Substituting in $ab = 0.04$, $(0.5 - b)b = 0.04$ (1 mark)

$\Rightarrow 0.5b - b^2 = 0.04 \Rightarrow b^2 - 0.5b + 0.04 = 0$ (1 mark)

$\Rightarrow b = 0.4, 0.1$

$b = 0.4 \Rightarrow a = \frac{0.04}{0.4} = 0.1$

$b = 0.1 \Rightarrow a = \frac{0.04}{0.1} = 0.4$ (2 marks)

Given $a > b$, $a = 0.4$, $b = 0.1$

(d) If X & Y are independent, $P(X \cap Y) = P(X) \times P(Y)$

$P(X \cap Y) = a = 0.4$ $P(X) = 0.3 + 0.4 = 0.7$ $P(Y) = 0.2 + 0.4 = 0.6$
 $P(X) \times P(Y) = 0.7 \times 0.6 = 0.42 \neq 0.4$, so X & Y are not independent (2 marks)