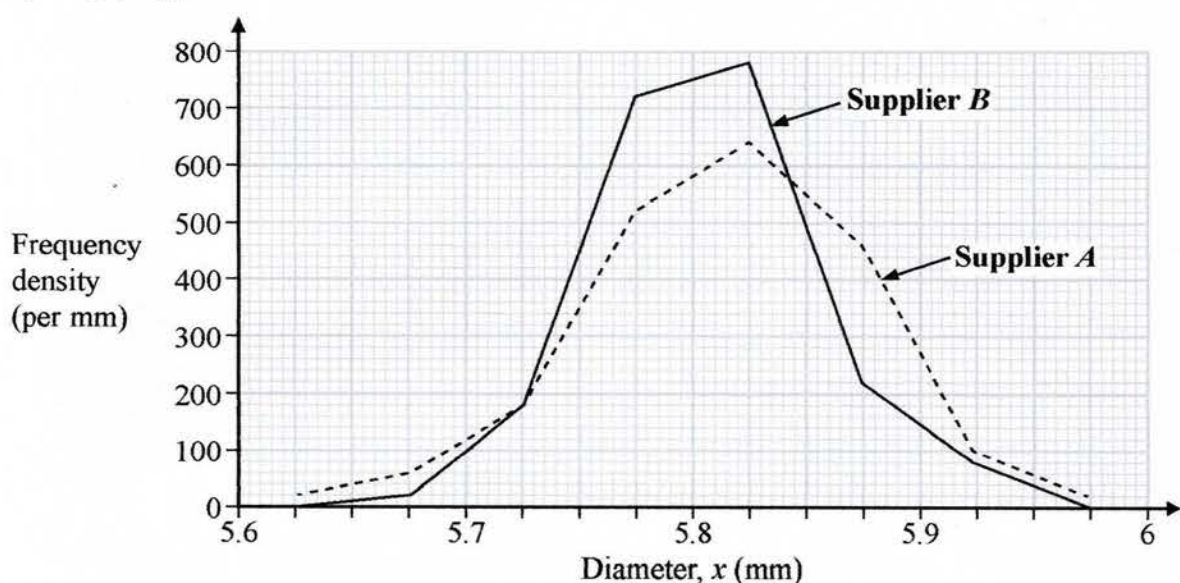**5.** An engineering company buys components with a specified diameter of 5.8 mm
Only components with a diameter between 5.75 mm and 5.85 mm are suitable for use.

A random sample of 100 of these components is taken from each of supplier A and
supplier B.
The diameters are grouped into 0.05 mm intervals and the results are shown in the
frequency polygons below.

Sam believes that supplier B will supply components with a more consistent diameter.

(a) Explain how the frequency polygons support Sam's belief.

(1)

(a) There is less variation for Supplier B.
More of their components fall in the acceptable range. (1 mark)

The modal class for each supplier is $5.80 \leqslant x < 5.85$

(b) Determine how many more components are in this class for supplier B than for
supplier A in these samples.

(2)

(b) frequency density = $\frac{\text{frequency}}{\text{class width}}$

$\Rightarrow$ frequency = frequency density $\times$ class width

For the modal class, 5.80 to 5.85,
For Supplier B, frequency = $780 \times 0.05 = 39$
For Supplier A, frequency = $640 \times 0.05 = 32$
Supplier B has $39 - 32 = 7$ more components here (2 marks)

Sam also finds these summary statistics for the sample from supplier B

$$\sum x^2 = 3371.1975 \qquad \sigma = 0.0483632$$

(c) Determine the mean for this sample.

(2)

(c) Variance = mean of squares - square of mean
$\sigma^2 = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2$

$0.0483632^2 = \frac{3371.1975}{100} - \bar{x}^2$

$\Rightarrow \bar{x} = \sqrt{\frac{3371.1975}{100} - 0.0483632^2} = 5.806$ mm (2 marks)

The diameter of components, D mm, from supplier B may be modelled by

$$D \sim N(5.81, 0.048^2)$$

Any component with a diameter less than 5.75 mm is not used.

(d) Using the model, find the probability that a component randomly selected from
supplier B is not used.

(1)

(d) $D \sim N(5.81, 0.048^2)$
$P(D < 5.75) = 0.105649... = 0.106$ 3sf (1 mark)

(fx-991EX: MENU 7 - Dist / Normal CD
fx: CG50 : MENU 2 - Stats / DIST / NORM / Ncd)

Any component with a diameter more than 5.85 mm is machined down by the company
and is then suitable for use.

(e) In a random sample of 40 components from supplier B that are suitable for use,
estimate how many had been machined down by the company.

(4)

(e) $P(\text{machined down} \mid \text{accepted}) = P(D > 5.85 \mid D > 5.75)$

$= \frac{P(D > 5.85)}{P(D > 5.75)}$ (if >5.85 then also >5.75) (1 mark) $= \frac{1 - 0.79767...}{1 - 0.105649...}$ from Calculator

$= 0.2262...$ (1 mark)

If each component has a $0.2262...$ chance of having been
machined down, in a sample of 40, we expect

$40 \times 0.2262... = 9.0491... \approx 9$
to have been machined down (2 marks)