Qu 5	Scheme	Marks	AO
(a)	$P(S \cap \{X = 50\}) = P(S \cap \{X = 80\})$[= a constant, V] $\Rightarrow b \times \frac{k}{50} = c \times \frac{k}{80}$		
	May see: $\frac{k}{50} = \frac{V}{b}$ and $\frac{k}{80} = \frac{V}{c}$ (condone any letter for V even S)	M1	3.1a
	So $\underline{c = \frac{8}{5}b}$ *	A1cso*	1.1b
		(2)	
(b)	$d = 2b$ **or** $a = \frac{2}{5}b$ **or** $c = 4a$ **or** $d = 5a$ **or** $d = \frac{5}{4}c$	M1 A1	2.1 3.3
	$\frac{2}{5}b + b + \frac{8}{5}b + 2b = 1$	M1	2.1
	$\Rightarrow 5b = 1$ so $b = \frac{1}{5}$ (o.e.)	A1	1.1b
	$\underline{\underline{a = \frac{2}{25} \quad b = \frac{1}{5} \quad c = \frac{8}{25} \quad d = \frac{2}{5}}}$	A1	3.2a
		(5)	
(c)	[Experiment suggests for Nav] $P(S \mid \{X = 100\}) = 0.3 \Rightarrow k = 30$		
	or $0.3 = \frac{V}{0.4} \Rightarrow V = 0.12$		
	So model won't work since	B1	2.4
	$P(S \mid X = 20) = \frac{30}{20}$ or $\frac{0.12}{0.08}$ and so would be greater than 1		
		(1)	
		(8 marks)	

Notes

(a) M1 for use of $P(S \mid X = x) \times P(X = x)$ for $x = 50$ and $x = 80$ (Must see k or their V)
Any expression or equation MUST be based on the probability statements in qu.

***** A1cso for rearranging to required result, no incorrect work seen, condone poor notation

NB Use of values e.g. $b = \frac{50}{20 + 50 + 80 + 100}$ to prove (a) is M0A0 but scores 2nd M1A1 in (b)

Marks for (b) may be awarded for work seen in (a)

(b) 1st M1 for at least one other relationship (either probability the subject) from the list.
1st A1 for a second different relationship (either probability the subject) from the list.

or Allow for: $\frac{ak}{20} = \frac{bk}{50} = \frac{ck}{80} = \frac{dk}{100}$ for 1st M1 1st A1

2nd M1 for using or stating sum of prob's = 1 May be implied by one correct probability.
2nd A1 for one correct probability e.g. $b = \frac{1}{5}$ or exact equivalent such as 0.2
3rd A1 for all correct probabilities. Allow exact equivalents e.g. $c = 0.32$
Sight of correct distribution or list of probs with no obvious incorrect working is 5/5

(c) B1 for deducing $k = 30$ **and** giving a suitable example to show model breaks down

Notes on Question 5

The question essentially uses the definition of $P(A \mid B)$ given in the formula booklet.

In particular $P\left(S \mid \{X = x\}\right) = \dfrac{P\left(S \cap \{X = x\}\right)}{P\left(X = x\right)}$ [1]

The first "blob" tells us that $P\left(S \mid \{X = x\}\right) = \dfrac{k}{x}$ where k is a constant.

The second "blob" tells us that $P\left(S \cap \{X = x\}\right)$ is the same for all x so $P\left(S \cap \{X = x\}\right) = V$ where V is a constant.

Using these results in [1] gives $\dfrac{k}{x} = \dfrac{V}{P(X = x)}$ [2]

Line 1 of MS for part (a) uses $V = P\left(X = x\right) \times \dfrac{k}{x}$ for $x = 50$ and $x = 80$

Line 2 of MS for part (a) uses [2] with $x = 50$ and $x = 80$

Other implications

Equation [1] can be rearranged to give $P\left(X = x\right) = x \times \dfrac{V}{k}$ [3]

So when $a + b + c + d = 1$ is used this gives $1 = \dfrac{V}{k}\left(20 + 50 + 80 + 100\right)$ or $\dfrac{V}{k} = \dfrac{1}{250}$ [4]

In particular if we use this relationship in [3] the probabilities a, b, c and d can simply be written down for example $b = \dfrac{50}{250}$ as given in the **NB** in the notes on the MS.

The point is that k and V will vary according to equation [4] but as part (c) shows there are some restrictions on the values k, and therefore V, can take.

Since $\dfrac{k}{x}$ is a probability then, ignoring the trivial cases*, $0 < \dfrac{k}{x} < 1$ and the "restricting" value of x is clearly $x = 20$ so $0 < k < 20$ and from [4] we get $0 < V < \dfrac{20}{250} = \dfrac{2}{25} = a$

So the restrictions on k and on V are given by the shortest distance and its associated probability.

* $k = 0$ would say Tisam can never get the ball in the cup no matter what the distance.

$k = 20$ says she always gets the ball in the cup for any distance.