

2. A machine fills packets with sweets and $\frac{1}{7}$ of the packets also contain a prize.

The packets of sweets are placed in boxes before being delivered to shops.
There are 40 packets of sweets in each box.

The random variable T represents the number of packets of sweets that contain a prize in each box.

(a) State a condition needed for T to be modelled by $B(40, \frac{1}{7})$

(a) packets must be placed in boxes independently of each other
so each packet has a constant probability of containing a prize. (1)

A box is selected at random.

(b) Using $T \sim B(40, \frac{1}{7})$ find

(i) the probability that the box has exactly 6 packets containing a prize,

(ii) the probability that the box has fewer than 3 packets containing a prize. (2)

$$(b)(i) P(T=6) = {}^{40}C_6 \left(\frac{1}{7}\right)^6 \left(\frac{6}{7}\right)^{34} = \left[\begin{array}{l} \text{(Calculator)} \\ \text{MENU Distribution} \\ \text{Binomial PD} \end{array} \right] = 0.1727... = 0.173 \text{ (3sf)}$$

$$(ii) P(T < 3) = P(T \leq 2) = \left[\begin{array}{l} \text{(Calculator)} \\ \text{MENU Distribution} \\ \text{Binomial CD} \end{array} \right] = 0.06158... = 0.0616 \text{ (3sf)}$$

Kamil's sweet shop buys 5 boxes of these sweets.

(c) Find the probability that exactly 2 of these 5 boxes have fewer than 3 packets containing a prize. (2)

(c) Let K be the number of boxes that contain fewer than 3 prize packets

$$K \sim B(5, 0.0616)$$

$$P(K=2) = {}^5C_2 (0.0616)^2 (1-0.0616)^3 = \left[\begin{array}{l} \text{(Calculator)} \\ \text{MENU Distribution} \\ \text{Binomial PD} \end{array} \right] = 0.03135... = 0.0314 \text{ (3sf)}$$

Kamil claims that the proportion of packets containing a prize is less than $\frac{1}{7}$

A random sample of 110 packets is taken and 9 packets contain a prize.

(d) Use a suitable test to assess Kamil's claim.

You should

- state your hypotheses clearly
- use a 5% level of significance

(4)

$$(d) T \sim B(110, p)$$

$$H_0: p = \frac{1}{7}$$

$$H_1: p < \frac{1}{7}$$

$$\text{Under } H_0, P(T \leq 9) = 0.03829... = 3.829... \%$$

Under H_0 the probability of 9 or fewer boxes containing a prize is less than 5%, so we can reject H_0 in favour of H_1 at the 5% significance level.

There is evidence to support Kamil's claim.