

5. Tisam is playing a game.
She uses a ball, a cup and a spinner.

The random variable X represents the number the spinner lands on when it is spun.
The probability distribution of X is given in the following table

x	20	50	80	100
$P(X=x)$	a	b	c	d

where a , b , c and d are probabilities.

To play the game

- the spinner is spun to obtain a value of x
- Tisam then stands x cm from the cup and tries to throw the ball into the cup

The event S represents the event that Tisam successfully throws the ball into the cup.

To model this game Tisam assumes that

- $P(S|X=x) = \frac{k}{x}$ where k is a constant
- $P(S \cap \{X=x\})$ should be the same whatever value of x is obtained from the spinner

Using Tisam's model,

- (a) show that $c = \frac{8}{5}b$ (2)

<u>(a)</u>	x	20	50	80	100
	$P(X=x)$	a	b	c	d
	$P(S X=x)$	$\frac{k}{20}$	$\frac{k}{50}$	$\frac{k}{80}$	$\frac{k}{100}$
	$P(S \cap \{X=x\})$	$\frac{ak}{20}$	$\frac{bk}{50}$	$\frac{ck}{80}$	$\frac{dk}{100}$
	$= P(S X=x) \times P(X=x)$				

Given $(S \cap \{X=x\})$ is the same for all x , $\frac{bk}{50} = \frac{ck}{80} \Rightarrow c = \frac{8}{5}b$

- (b) find the probability distribution of X (5)

(b) $a + b + c + d = 1$

$$\frac{ak}{20} = \frac{bk}{50} \Rightarrow a = \frac{2b}{5} \quad \frac{dk}{100} = \frac{bk}{50} \Rightarrow d = 2b$$

$$\text{So, } \frac{2b}{5} + b + \frac{8b}{5} + 2b = 1 \Rightarrow 5b = 1 \Rightarrow b = \frac{1}{5}$$

$$c = \frac{8(\frac{1}{5})}{5} = \frac{8}{25} \quad a = \frac{2(\frac{1}{5})}{5} = \frac{2}{25} \quad d = 2(\frac{1}{5}) = \frac{2}{5}$$

So, probability distribution is,

x	20	50	80	100
$P(X=x)$	$\frac{2}{25}$	$\frac{1}{5}$	$\frac{8}{25}$	$\frac{2}{5}$

Nav tries, a large number of times, to throw the ball into the cup from a distance of 100 cm.

He successfully gets the ball in the cup 30% of the time.

- (c) State, giving a reason, why Tisam's model of this game is not suitable to describe Nav playing the game for all values of X (1)

(c) For Nav, $P(S|X=100) = \frac{3}{10}$

$$\text{Tisam model assumes } P(S|\{X=x\}) = \frac{k}{x} \Rightarrow \frac{3}{10} = \frac{k}{100} \Rightarrow k = 30$$

$$\text{With } k=30, P(S|X=20) = \frac{30}{20} = 1.5 > 1, \text{ so impossible.}$$