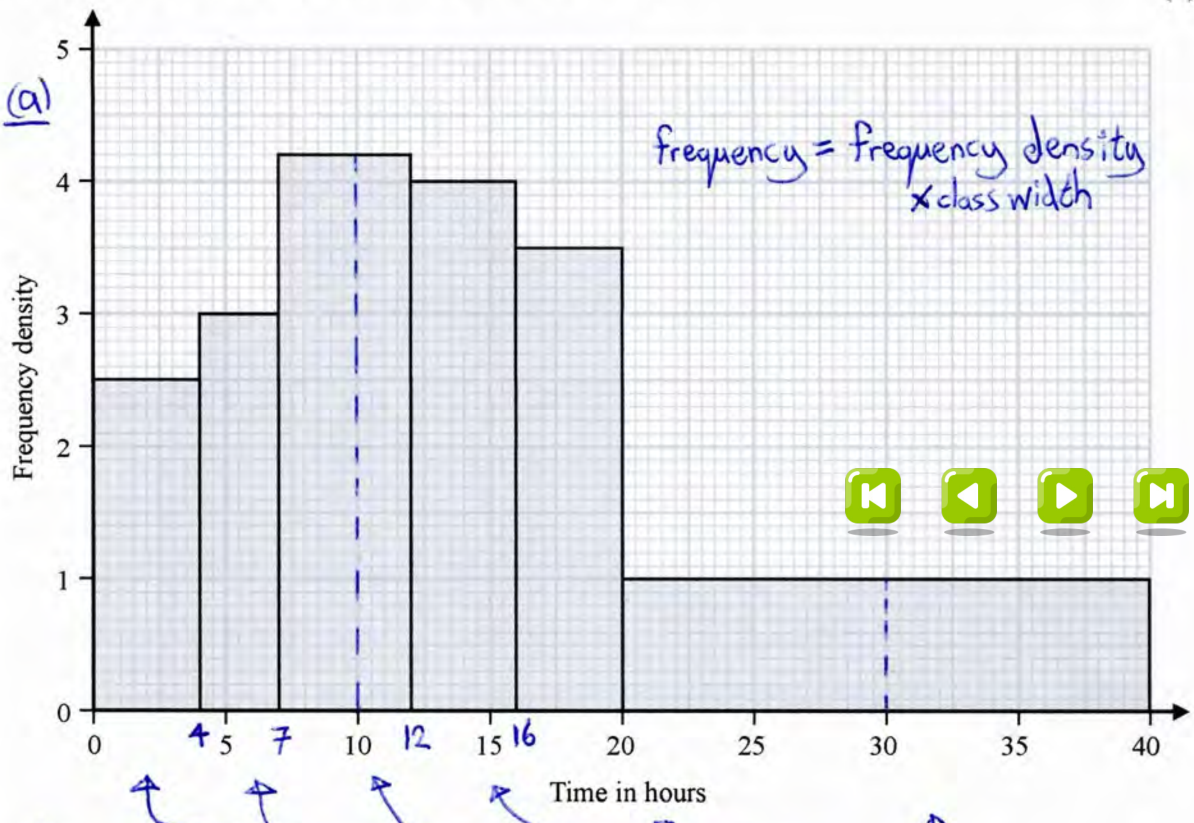
6. A medical researcher is studying the number of hours, T, a patient stays in hospital following a particular operation.

The histogram on the page opposite summarises the results for a random sample of 90 patients.

(a) Use the histogram to estimate $P(10 < T < 30)$ **(2)**



frequency

$2.5 \times 4 = 10$ $3 \times 3 = 9$ $4.2 \times 5 = 21$ $4 \times 4 = 16$ $3.5 \times 4 = 14$ $1 \times 20 = 20$

Total Frequency $= 10 + 9 + 21 + 16 + 14 + 20 = 90$

Frequency Between 10 & 30 $= (4.2 \times 2) + 16 + 14 + (1 \times 10) = 48.4$

$P(10 < T < 30) = \frac{48.4}{90} = 0.5377... = 0.538$ (3sf)

For these 90 patients the time spent in hospital following the operation had

- a mean of 14.9 hours
- a standard deviation of 9.3 hours

Tomas suggests that T can be modelled by $N(14.9, 9.3^2)$

(b) With reference to the histogram, state, giving a reason, whether or not Tomas' model could be suitable. **(1)**

(b) The histogram is not symmetric but skewed to the right. The Normal distribution is symmetric, so not suitable here.

Xiang suggests that the frequency polygon based on this histogram could be modelled by a curve with equation

$$y = kxe^{-x} \quad 0 \leqslant x \leqslant 4$$

where

- x is measured in **tens of hours**
- k is a constant

(c) Use algebraic integration to show that

$$\int_0^n xe^{-x}\,dx = 1 - (n+1)e^{-n}$$

(4)

(c) $\int_0^n x e^{-x}\,dx$ (use Integration by Parts $\int uv' = uv - \int u'v$)

$u = x$, $v' = e^{-x}$

$u' = 1$ $\quad v = -e^{-x}$

Definite Integral $= \left[x(-e^{-x})\right]_0^n - \int_0^n (1)(-e^{-x})$

$= \left[-xe^{-x}\right]_0^n + \int_0^n e^{-x}\,dx$

$= \left[-xe^{-x}\right]_0^n + \left[-e^{-x}\right]_0^n$

$= -ne^{-n} - 0 + (-e^{-n} - (-e^{0}))$

$= -ne^{-n} - e^{-n} + 1 = 1 - (n+1)e^{-n}$

(d) Show that, for Xiang's model, $k = 99$ to the nearest integer. **(3)**

(d) To model total frequency of 90,

$\int_0^n kxe^{-x} = 90$ $\quad n = \frac{40}{10} = 4$, because x measured in tens of hours

$\Rightarrow k\int_0^4 xe^{-x} = 90$

$\Rightarrow k(1 - (4+1)e^{-4}) = 90$

$k(1 - 5e^{-4}) = 90$

$k = \frac{90}{1 - 5e^{-4}} = 99.0729...$

$= 99$ (to nearest integer)

(e) Estimate $P(10 < T < 30)$ using

(i) Tomas' model of $T \sim N(14.9, 9.3^2)$ **(1)**

(ii) Xiang's curve with equation $y = 99xe^{-x}$ and the answer to part (c) **(2)**

(e)(i) If $T \sim N(14.9, 9.3^2)$, $P(10 < T < 30) = \left[\begin{array}{l}\text{(Calculator)} \\ \text{MENU Distribution} \\ \text{Normal CD} \\ \text{Lower: 10 / Upper: 30}\end{array}\right] = 0.6486... = 0.649$ (3sf)

(ii) Using Xiang's model, $P(10 < T < 30)$

$= \frac{\int_1^3 99xe^{-x}}{90} = \frac{99}{90}\left[(1 - (x+1)e^{-x})\right]_1^3$

$= \frac{99}{90}\left((1 - 4e^{-3}) - (1 - 2e^{-1})\right) = 0.5902... = 0.590$ (3sf)

The researcher decides to use Xiang's curve to model $P(a < T < b)$

(f) State one limitation of Xiang's model. **(1)**

(f) Patients could stay in hospital longer than 40 hours following their operation. Xiang's model would not allow for this, while the Normal model would.