9.

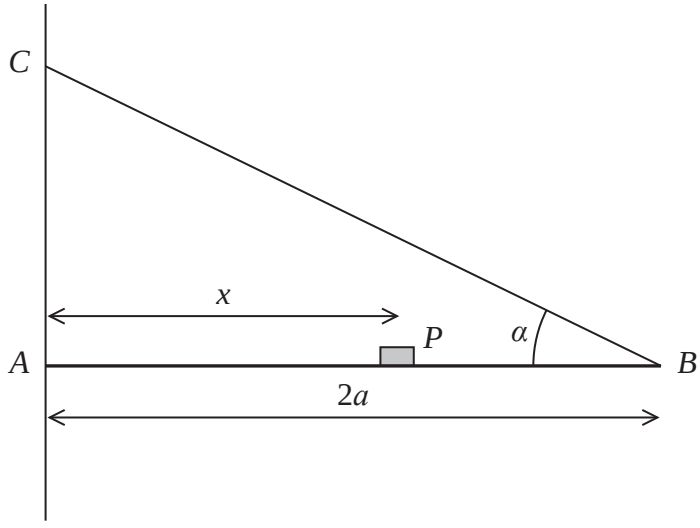

Figure 3

A plank, AB, of mass M and length $2a$, rests with its end A against a rough vertical wall. The plank is held in a horizontal position by a rope. One end of the rope is attached to the plank at B and the other end is attached to the wall at the point C, which is vertically above A.

A small block of mass $3M$ is placed on the plank at the point P, where $AP = x$. The plank is in equilibrium in a vertical plane which is perpendicular to the wall.

The angle between the rope and the plank is α, where $\tan\alpha = \frac{3}{4}$, as shown in Figure 3.

The plank is modelled as a uniform rod, the block is modelled as a particle and the rope is modelled as a light inextensible string.

(a) Using the model, show that the tension in the rope is $\frac{5Mg(3x + a)}{6a}$

(3)

The magnitude of the horizontal component of the force exerted on the plank at A by the wall is $2Mg$.

(b) Find x in terms of a.

(2)

The force exerted on the plank at A by the wall acts in a direction which makes an angle β with the horizontal.

(c) Find the value of $\tan\beta$

(5)

The rope will break if the tension in it exceeds $5Mg$.

(d) Explain how this will restrict the possible positions of P. You must justify your answer carefully.

(3)