

2. (Friction, F_r , opposes motion. Given A moves down plane, F_r must act up plane)

$$\tan \alpha = \frac{3}{4}$$

$$\Rightarrow \sin \alpha = \frac{3}{\sqrt{3^2+4^2}} = \frac{3}{5}$$

$$\Rightarrow \cos \alpha = \frac{4}{\sqrt{3^2+4^2}} = \frac{4}{5}$$

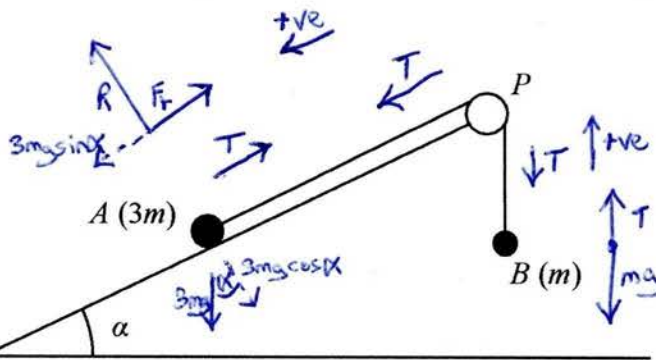


Figure 1

A small stone A of mass $3m$ is attached to one end of a string.

A small stone B of mass m is attached to the other end of the string.

Initially A is held at rest on a fixed rough plane.

The plane is inclined to the horizontal at an angle α , where $\tan \alpha = \frac{3}{4}$

The string passes over a pulley P that is fixed at the top of the plane.

The part of the string from A to P is parallel to a line of greatest slope of the plane.

Stone B hangs freely below P , as shown in Figure 1.

The coefficient of friction between A and the plane is $\frac{1}{6}$

Stone A is released from rest and begins to move down the plane.

The stones are modelled as particles.

The pulley is modelled as being small and smooth.

The string is modelled as being light and inextensible.

(a) $F = ma$

$$3mg \sin \alpha - T - F_r = 3ma \quad (2 \text{ marks})$$

(b) In motion, $F_r = \mu R$

$$= \left(\frac{1}{6}\right) 3mg \cos \alpha = \frac{1}{2} mg \cos \alpha \quad (3 \text{ marks})$$

so,

$$3mg \sin \alpha - T - \frac{1}{2} mg \cos \alpha = 3ma$$

$$3mg \left(\frac{3}{5}\right) - T - \frac{1}{2} mg \left(\frac{4}{5}\right) = 3ma$$

$$\frac{7}{5} mg - T = 3ma$$

Now, we need equation of motion for B to eliminate T :

$$T - mg = ma \quad (\text{same acceleration})$$

$$\Rightarrow T = mg + ma \quad (2 \text{ marks})$$

Substituting for T ,

$$\frac{7}{5} mg - (mg + ma) = 3ma$$

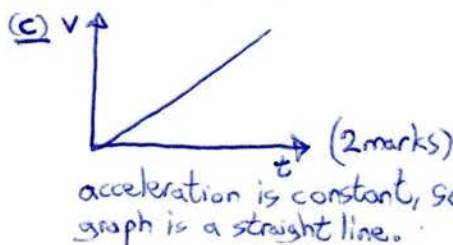
$$\Rightarrow \frac{2}{5} mg - ma = 3ma$$

$$\frac{2}{5} mg = 4ma \Rightarrow a = \frac{1}{10} g \quad (2 \text{ marks})$$

Using the model for the motion of the system before B reaches the pulley,

(a) write down an equation of motion for A

(b) show that the acceleration of A is $\frac{1}{10} g$



(c) sketch a velocity-time graph for the motion of B , from the instant when A is released from rest to the instant just before B reaches the pulley, explaining your answer.

(d) The tensions on both sides of the pulley would be different, because different lengths of string would have different weights. (1 mark)

In reality, the string is not light.

(d) State how this would affect the working in part (b).