

3.

(Reactions act perpendicular to the surface exerting the Reaction)

(beam is "uniform" so its weight acts at its middle)

kmg

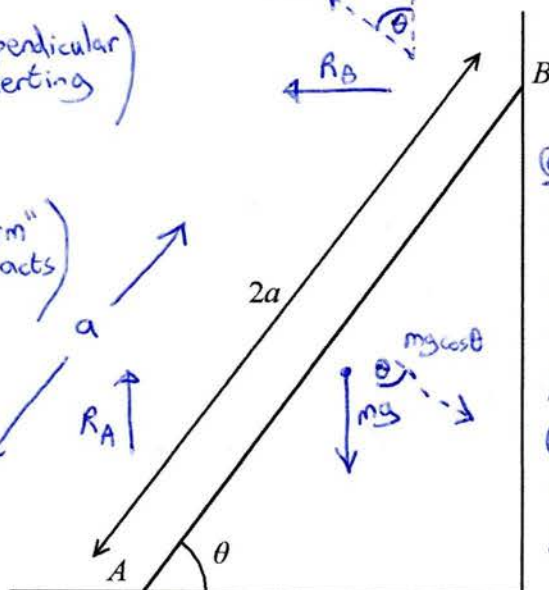


Figure 2

(a) Resolving vertically,
 $R(\uparrow): R_A = mg$ (1 mark)

Resolving horizontally,
 $R(\leftarrow): R_B = F_r$

Resolving Moments about A
 (so we don't need to worry about R_A & F_r)

$$M(A): mg \cos \theta \times a = R_B \sin \theta \times 2a \quad (2 \text{ marks})$$

$$\Rightarrow R_B = \frac{mg \cos \theta}{2 \sin \theta} = \frac{mg}{2} \cot \theta$$

$$\Rightarrow F_r = R_B = \frac{mg}{2} \cot \theta$$

A beam AB has mass m and length $2a$.

The beam rests in equilibrium with A on rough horizontal ground and with B against a smooth vertical wall.

The beam is inclined to the horizontal at an angle θ , as shown in Figure 2.

The coefficient of friction between the beam and the ground is μ

The beam is modelled as a uniform rod resting in a vertical plane that is perpendicular to the wall.

$$F_r \leq \mu R_A$$

$$\frac{mg}{2} \cot \theta \leq \mu mg$$

$$\Rightarrow \mu \geq \frac{1}{2} \cot \theta \quad (2 \text{ marks})$$

Using the model,

(a) show that $\mu \geq \frac{1}{2} \cot \theta$

(b) "in limiting equilibrium" ladder must be on point of sliding
 F_r could not be towards wall, because it is at a max, but would previously have been more without kmg supporting it.
 So F_r is now away from wall $\leftarrow F_r$

$$R(\leftarrow): kmg = F_r + R_B \quad \text{Nothing has changed vertically so still, } R_A = mg \quad (5)$$

A horizontal force of magnitude kmg , where k is a constant, is now applied to the beam at A.

This force acts in a direction that is perpendicular to the wall and towards the wall.

Given that $\tan \theta = \frac{5}{4}$, $\mu = \frac{1}{2}$ and the beam is now in limiting equilibrium,

(b) use the model to find the value of k .

(b) contd

$$M(A): \text{Nothing has changed so still, } R_B = \frac{mg}{2} \cot \theta$$

(5)

$$kmg = F_r + R_B \Rightarrow kmg = \mu R_A + R_B$$

$$\Rightarrow kmg = \mu mg + \frac{mg}{2} \cot \theta \quad \text{Given } \tan \theta = \frac{5}{4}, \mu = \frac{1}{2} \quad kmg = \frac{1}{2}mg + \frac{mg}{2} \left(\frac{4}{5}\right) \Rightarrow k = \frac{9}{10} \quad (5 \text{ marks})$$