

5. At time  $t$  seconds, a particle  $P$  has velocity  $\mathbf{v} \text{ ms}^{-1}$ , where

(a)  $\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d}{dt} \begin{pmatrix} 3t^{\frac{1}{2}} \\ -2t \end{pmatrix} = \begin{pmatrix} \frac{3}{2}t^{-\frac{1}{2}} \\ -2 \end{pmatrix}$   $\mathbf{v} = 3t^{\frac{1}{2}} \mathbf{i} - 2t \mathbf{j} \quad t > 0$  (2 marks)

(a) Find the acceleration of  $P$  at time  $t$  seconds, where  $t > 0$

(b) in direction  $\mathbf{j} - \mathbf{i}$ ,  $\mathbf{i}$  component =  $-\mathbf{j}$  component  $\Rightarrow 3t^{\frac{1}{2}} = -(-2t)$  (1 mark) (2)

(b) Find the value of  $t$  at the instant when  $P$  is moving in the direction of  $\mathbf{i} - \mathbf{j}$

(b) ~~cota~~  $2t - 3t^{\frac{1}{2}} = 0 \Rightarrow t^{\frac{1}{2}}(2t^{\frac{1}{2}} - 3) = 0 \Rightarrow t = \cancel{0}, \frac{9}{4} \text{ s (given } t > 0)$  (2 marks) (3)

At time  $t$  seconds, where  $t > 0$ , the position vector of  $P$ , relative to a fixed origin  $O$ , is  $\mathbf{r}$  metres.

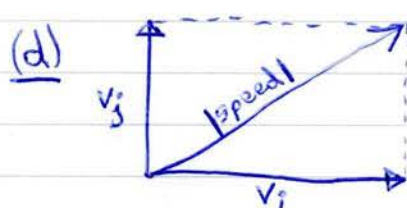
(c)  $\mathbf{r} = \int \mathbf{v} dt = \int \begin{pmatrix} 3t^{\frac{1}{2}} \\ -2t \end{pmatrix} dt = \begin{pmatrix} 2t^{\frac{3}{2}} + c_1 \\ -t^2 + c_2 \end{pmatrix}$  (2 marks)

When  $t = 1$ ,  $\mathbf{r} = -\mathbf{j}$

(c) Find an expression for  $\mathbf{r}$  in terms of  $t$ . When  $t = 1$ ,  $\mathbf{r} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 2(1)^{\frac{3}{2}} + c_1 \\ -(1)^2 + c_2 \end{pmatrix} = \begin{pmatrix} 2 + c_1 \\ -1 + c_2 \end{pmatrix}$  (3)

(d) Find the exact distance of  $P$  from  $O$  at the instant when  $P$  is moving with speed  $10 \text{ ms}^{-1}$

(c) ~~cota~~  $\Rightarrow \mathbf{r} = \begin{pmatrix} 2t^{\frac{3}{2}} - 2 \\ -t^2 \end{pmatrix}$  (1 mark) (6)



$$\text{speed} = 10 = \sqrt{(3t^{\frac{1}{2}})^2 + (-2t)^2} \quad (1 \text{ mark})$$

$$= \sqrt{9t + 4t^2}$$

$$\Rightarrow 100 = 9t + 4t^2 \Rightarrow 4t^2 + 9t - 100 = 0 \quad (1 \text{ mark})$$

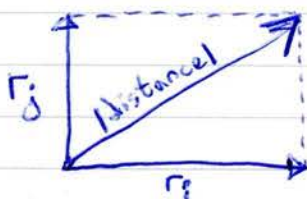
Can solve quadratic using Calculator

fx-991EX: MENU A / Polynomial / Degree 2 ...

fx-CG50: MENU A / Polynomial / Degree 2 ...

$$\Rightarrow t = \cancel{\frac{-9}{8}}, 4 \text{ s (1 mark)}$$

when  $t = 4$ ,  $\mathbf{r} = \begin{pmatrix} 2(4)^{\frac{3}{2}} - 2 \\ -(4)^2 \end{pmatrix} = \begin{pmatrix} 14 \\ -16 \end{pmatrix}$  (1 mark)



$$\text{distance} = \sqrt{14^2 + (-16)^2} \quad (1 \text{ mark})$$

$$= \sqrt{196 + 256}$$

$$= 2\sqrt{113} \text{ m.} \quad (1 \text{ mark})$$