

2.

[In this question \mathbf{i} and \mathbf{j} are horizontal unit vectors.]A particle P of mass 0.5 kg is moving on a smooth horizontal plane.The origin O is on the plane.At time $t = 0$, P passes through O moving with velocity $(\mathbf{i} - \mathbf{j}) \text{ m s}^{-1}$ At time t seconds, the resultant horizontal force acting on P is

$$[(3t - 1)\mathbf{i} + 2\mathbf{j}] \text{ N}$$

(a) Find the velocity of P at $t = 2$

(5)

(b) Find the distance of P from O at $t = 2$

(4)

(a) $F = ma$

$$\begin{pmatrix} 3t-1 \\ 2 \end{pmatrix} = 0.5 \begin{pmatrix} a_i \\ a_j \end{pmatrix} \quad (1 \text{ mark}) \Rightarrow \begin{pmatrix} a_i \\ a_j \end{pmatrix} = \begin{pmatrix} 6t-2 \\ 4 \end{pmatrix}$$

$$v = \int a \, dt = \int \begin{pmatrix} 6t-2 \\ 4 \end{pmatrix} dt = \begin{pmatrix} 3t^2-2t+c_i \\ 4t+c_j \end{pmatrix} \quad (2 \text{ marks})$$

$$\text{at } t=0, v = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \text{ so } \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 3(0)^2-2(0)+c_i \\ 4(0)+c_j \end{pmatrix} \Rightarrow \begin{matrix} c_i = 1 \\ c_j = -1 \end{matrix}$$

$$\text{so, } v = \begin{pmatrix} 3t^2-2t+1 \\ 4t-1 \end{pmatrix} \quad \text{At } t=2, v = \begin{pmatrix} 3(2)^2-2(2)+1 \\ 4(2)-1 \end{pmatrix} = \begin{pmatrix} 9 \\ 7 \end{pmatrix} \text{ m s}^{-1} \quad (2 \text{ marks})$$

(b) $s = \int v \, dt = \int \begin{pmatrix} 3t^2-2t+1 \\ 4t-1 \end{pmatrix} dt = \begin{pmatrix} t^3-t^2+t+k_i \\ 2t^2-t+k_j \end{pmatrix} \quad (2 \text{ marks})$

$$\text{at } t=0, s = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \text{ so } \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0^3-0^2+0+k_i \\ 2(0)^2-0+k_j \end{pmatrix} \Rightarrow \begin{matrix} k_i = 0 \\ k_j = 0 \end{matrix}$$

$$\text{so, } s = \begin{pmatrix} t^3-t^2+t \\ 2t^2-t \end{pmatrix} \quad \text{At } t=2, s = \begin{pmatrix} 2^3-2^2+2 \\ 2(2)^2-2 \end{pmatrix} = \begin{pmatrix} 6 \\ 6 \end{pmatrix} \text{ m}$$

$$\text{distance} = |s| = \sqrt{6^2 + 6^2} = \sqrt{72} \text{ m} \quad (2 \text{ marks})$$