

4.

[In this question the unit vectors \mathbf{i} and \mathbf{j} are directed horizontally and vertically upwards respectively.]

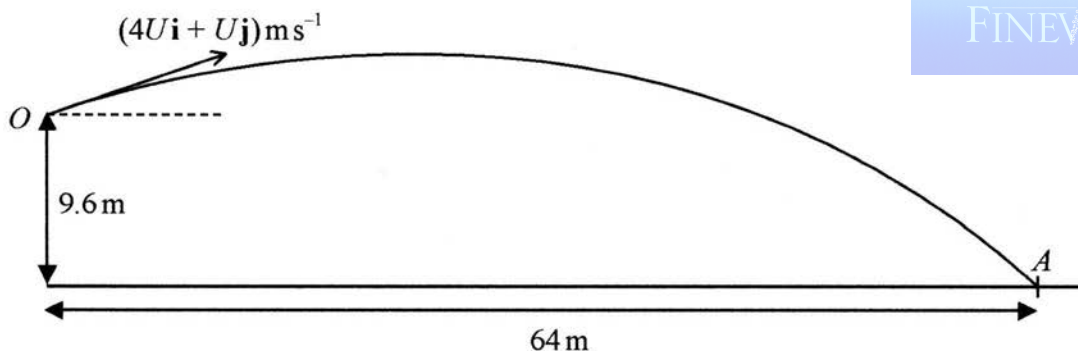


Figure 2

The point O is 9.6 m above horizontal ground.

A small ball is projected with velocity $(4U\mathbf{i} + U\mathbf{j})\text{ms}^{-1}$, where U is a positive constant, from the point O

The ball first hits the ground T seconds later, at the point A

The point A is at a horizontal distance of 64 m from O , as shown in Figure 2.

In an initial model (a) Resolving horizontally, there is no acceleration, so $s = ut$
 $64 = 4U \times T$ ($4U$ is the horizontal (\mathbf{i}) component) $\Rightarrow UT = \frac{64}{4} = 16$ (2 marks)

- the ball is modelled as a particle moving under gravity
- air resistance is ignored
- the ball has an initial speed of $V\text{ms}^{-1}$

Using this model,

(a) show that $UT = 16$

(b) find the value of V

(c) State two improvements to the model, other than including air resistance, that would make the model more realistic.

(b) Resolving vertically, $s = ut + \frac{1}{2}at^2$
 with +ve direction \uparrow , when ball lands,
 $s = -9.6$ $t = T$ $a = -g = -9.8$ $u = U$ (vertical component)
 so, $-9.6 = UT + \frac{1}{2}(-9.8)T^2$ (1 mark)
 $\Rightarrow 4.9T^2 - UT - 9.6 = 0$ (1 mark)

(b) contd $UT = 16$, so $4.9T^2 - 16 - 9.6 = 0 \Rightarrow T^2 = \frac{25.6}{4.9}$ (2)

$T = \frac{16}{7}$ $U = \frac{16}{T} = 7$ (2 marks)

Speed, $V = \sqrt{u^2 + (4u)^2} = \sqrt{7^2 + 28^2} = 28.86 = 28.9\text{ms}^{-1}$ (2 marks) (6)

(2)

(c) allow for wind effects / spin of the ball / use a more accurate value for g
 (2 marks)