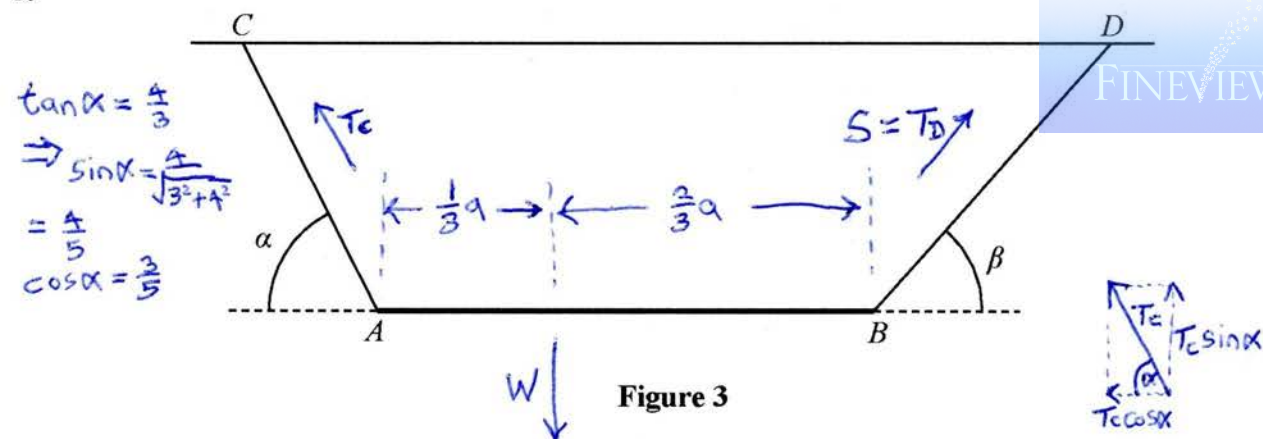


5.



A non-uniform rod  $AB$  is held in equilibrium in a horizontal position by two light inextensible strings.

The first string has one end attached to the end  $A$  of the rod and the other end attached to a point  $C$  on a horizontal ceiling.

The second string has one end attached to the end  $B$  of the rod and the other end attached to a point  $D$  on the horizontal ceiling, as shown in Figure 3.

The points  $A$ ,  $B$ ,  $C$  and  $D$  all lie in the same vertical plane.

Given that

- the rod  $AB$  has weight  $W$  and length  $a$
- the centre of mass of the rod is a distance  $\frac{1}{3}a$  from  $A$
- the string  $AC$  makes an angle  $\alpha$  with the horizontal, where  $\tan \alpha = \frac{4}{3}$
- the string  $BD$  makes an angle  $\beta$  with the horizontal

(a) show that the tension in the string  $AC$  is  $\frac{5}{6}W$

(b) Show that  $S \sin \beta = \frac{1}{3}W$

The tension in the string  $BD$  is  $S$

(c) Find  $S$  in terms of  $W$

(a) We don't know anything about angle  $\beta$ . If we take moments about  $B$ , we don't need to know anything about  $\beta$  (1 mark)

For equilibrium,

$$M(\odot) = M(\ominus)$$

$$T_C \sin \alpha (a) = W \left( \frac{2}{3}a \right)$$

$$T_C \left( \frac{4}{5} \right) a = W \frac{2}{3} a$$

$$T_C = \left( \frac{2}{3} \right) \left( \frac{5}{4} \right) W = \frac{5}{6} W$$

(2 marks)

(3)

(b)  $S \sin \beta$  is a vertical component.

$$R(\uparrow): T_C \sin \alpha + S \sin \beta = W$$

$$\frac{5}{6} W \left( \frac{4}{5} \right) + S \sin \beta = W$$

$$S \sin \beta = W - \frac{2}{3} W = \frac{1}{3} W \quad (2 \text{ marks})$$

(2)

(c) Find  $S$  in terms of  $W$

(c) We have resolved moments and vertically, but not horizontally.

$$\text{Resolving horizontally, } R(\leftrightarrow): T_C \cos \alpha = S \cos \beta \Rightarrow \frac{5}{6} W \left( \frac{3}{5} \right) = S \cos \beta = \frac{1}{2} W \quad (3 \text{ marks})$$

(5)

we have,

$$\begin{cases} S \sin \beta = \frac{1}{3} W \\ S \cos \beta = \frac{1}{2} W \end{cases} \Rightarrow \begin{cases} \sin \beta = \frac{W}{3S} \\ \cos \beta = \frac{W}{2S} \end{cases} \Rightarrow \sin^2 \beta + \cos^2 \beta = 1 \Rightarrow \frac{W^2}{9S^2} + \frac{W^2}{4S^2} = 1$$

$$\Rightarrow \frac{13W^2}{36S^2} = 1 \Rightarrow 13W^2 = 36S^2 \Rightarrow S = \frac{\sqrt{13}}{6} W \quad (2 \text{ marks})$$