

1.

[In this question, position vectors are given relative to a fixed origin.]

At time t seconds, where $t > 0$, a particle P has velocity $\mathbf{v} \text{ m s}^{-1}$ where

$$\mathbf{v} = 3t^2 \mathbf{i} - 6t^{\frac{1}{2}} \mathbf{j}$$

(a) Find the speed of P at time $t = 2$ seconds.

(2)

(b) Find an expression, in terms of t , \mathbf{i} and \mathbf{j} , for the acceleration of P at time t seconds, where $t > 0$

(2)

At time $t = 4$ seconds, the position vector of P is $(\mathbf{i} - 4\mathbf{j}) \text{ m}$.(c) Find the position vector of P at time $t = 1$ second.

(4)

$$\underline{\text{(a)}} \quad \mathbf{v} = \begin{pmatrix} 3t^2 \\ -6t^{\frac{1}{2}} \end{pmatrix} \quad \mathbf{v}(2) = \begin{pmatrix} 3(2)^2 \\ -6(2)^{\frac{1}{2}} \end{pmatrix} = \begin{pmatrix} 12 \\ -6\sqrt{2} \end{pmatrix}$$

$$\text{speed}_2 = |\mathbf{v}_2| = \sqrt{12^2 + (-6\sqrt{2})^2} \quad (1 \text{ mark})$$

$$= \sqrt{144 + 72} = \sqrt{216} = 6\sqrt{6} \text{ m s}^{-1} \quad (1 \text{ mark})$$

$$\underline{\text{(b)}} \quad \mathbf{a} = \frac{d\mathbf{v}}{dt} = \begin{pmatrix} \frac{d(3t^2)}{dt} \\ \frac{d(-6t^{\frac{1}{2}})}{dt} \end{pmatrix} = \begin{pmatrix} 6t \\ -3t^{-\frac{1}{2}} \end{pmatrix} \text{ m s}^{-2} \quad (2 \text{ marks})$$

$$\underline{\text{(c)}} \quad \mathbf{s} = \int \mathbf{v} dt = \begin{pmatrix} \int 3t^2 dt \\ \int -6t^{\frac{1}{2}} dt \end{pmatrix} = \begin{pmatrix} t^3 + c_1 \\ -4t^{\frac{3}{2}} + c_2 \end{pmatrix} \quad (2 \text{ marks})$$

$$\text{at } t = 4, \mathbf{s}(4) = \begin{pmatrix} 1 \\ -4 \end{pmatrix} \text{ so } \begin{pmatrix} 4^3 + c_1 \\ -4(4)^{\frac{3}{2}} + c_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -4 \end{pmatrix} \quad (1 \text{ mark})$$

$$4^3 + c_1 = 1 \Rightarrow c_1 = -63$$

$$-4(4)^{\frac{3}{2}} + c_2 = -4 \Rightarrow c_2 = 28$$

$$\text{so } \mathbf{s} = \begin{pmatrix} t^3 - 63 \\ -4t^{\frac{3}{2}} + 28 \end{pmatrix}$$

$$\mathbf{s}(1) = \begin{pmatrix} (1)^3 - 63 \\ -4(1)^{\frac{3}{2}} + 28 \end{pmatrix}$$

$$= \begin{pmatrix} -62 \\ 24 \end{pmatrix} \text{ m} \quad (1 \text{ mark})$$