

3.

[In this question, \mathbf{i} and \mathbf{j} are horizontal unit vectors.]

A particle P of mass 4 kg is at rest at the point A on a smooth horizontal plane.

At time $t = 0$, two forces, $\mathbf{F}_1 = (4\mathbf{i} - \mathbf{j})\text{ N}$ and $\mathbf{F}_2 = (\lambda\mathbf{i} + \mu\mathbf{j})\text{ N}$, where λ and μ are constants, are applied to P

Given that P moves in the direction of the vector $(3\mathbf{i} + \mathbf{j})$

(a) show that

$$\lambda - 3\mu + 7 = 0 \quad (4)$$

At time $t = 4$ seconds, P passes through the point B .

Given that $\lambda = 2$

(b) find the length of AB .

(5)

(a) Total Force on P ,

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = \begin{pmatrix} 4 \\ -1 \end{pmatrix} + \begin{pmatrix} \lambda \\ \mu \end{pmatrix} = \begin{pmatrix} 4 + \lambda \\ \mu - 1 \end{pmatrix} \text{ N} \quad (1 \text{ mark})$$

Because P starts at rest, acceleration is in the direction $\begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 3k \\ k \end{pmatrix}$ for some scalar k

$$\mathbf{F} = m\mathbf{a} \Rightarrow \begin{pmatrix} 4 + \lambda \\ \mu - 1 \end{pmatrix} = 4 \begin{pmatrix} 3k \\ k \end{pmatrix} \Rightarrow \begin{aligned} 4 + \lambda &= 12k \Rightarrow k = \frac{4 + \lambda}{12} \\ \mu - 1 &= 4k \Rightarrow k = \frac{\mu - 1}{4} \end{aligned} \quad (1 \text{ mark})$$

$$\text{so, } \frac{4 + \lambda}{12} = \frac{\mu - 1}{4} \Rightarrow \begin{aligned} 16 + 4\lambda &= 12\mu - 12 \\ 4 + \lambda &= 3\mu - 3 \\ \lambda - 3\mu + 7 &= 0 \end{aligned} \quad (2 \text{ marks})$$

(b) Given $\lambda = 2$, $2 - 3\mu + 7 = 0 \Rightarrow \mu = 3$ (1 mark)

$$\mathbf{F} = \begin{pmatrix} 4 + 2 \\ 3 - 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \end{pmatrix} = m\mathbf{a} = 4 \begin{pmatrix} 3k \\ k \end{pmatrix} = \begin{pmatrix} 12k \\ 4k \end{pmatrix} \quad (1 \text{ mark})$$

 so, $k = \frac{1}{2}$, $\mathbf{a} = \begin{pmatrix} \frac{3}{2} \\ \frac{1}{2} \end{pmatrix}$

$$s = ut + \frac{1}{2}at^2, s(4) = \begin{pmatrix} 0 \\ 0 \end{pmatrix} 4 + \frac{1}{2} \begin{pmatrix} \frac{3}{2} \\ \frac{1}{2} \end{pmatrix} 4^2 = \begin{pmatrix} 12 \\ 4 \end{pmatrix} \text{ m.} \quad (1 \text{ mark})$$

$$\text{distance } AB = \text{distance from } \begin{pmatrix} 0 \\ 0 \end{pmatrix} \text{ to } \begin{pmatrix} 12 \\ 4 \end{pmatrix} = \sqrt{12^2 + 4^2} = \sqrt{160} = 4\sqrt{10} \text{ m} \quad (2 \text{ marks})$$