



Figure 2

A uniform rod AB has mass M and length $2a$

A particle of mass $2M$ is attached to the rod at the point C , where $AC = 1.5a$

The rod rests with its end A on rough horizontal ground.

The rod is held in equilibrium at an angle θ to the ground by a light string that is attached to the end B of the rod.

The string is perpendicular to the rod, as shown in Figure 2.

- (a) Explain why the frictional force acting on the rod at A acts horizontally to the right on the diagram.

(a) Horizontal component of T acts to the left. Friction F_r is the only other force with a horizontal component, so it must act to the right, for equilibrium. (1 mark)

The tension in the string is T

- (b) Show that $T = 2Mg \cos \theta$

(b) Taking moments about A , for equilibrium, $M(\odot) = M(\circlearrowright)$
 $Mg \cos \theta \times a + 2Mg \cos \theta \times \frac{3}{2}a = T \times 2a$ (1 mark)
 $4Mg \cos \theta = 2T$
 $T = 2Mg \cos \theta$ (2 marks)

Given that $\cos \theta = \frac{3}{5}$

- (c) show that the magnitude of the vertical force exerted by the ground on the rod at A is $\frac{57Mg}{25}$

(c) Resolving vertically, for equilibrium, $\uparrow = \downarrow$
 $R + T \cos \theta = Mg + 2Mg = 3Mg$ (2 marks)
 Given $T = 2Mg \cos \theta$ & $\cos \theta = \frac{3}{5}$,

The coefficient of friction between the rod and the ground is μ

Given that the rod is in limiting equilibrium,

- (d) show that $\mu = \frac{8}{19}$

(d) Resolving horizontally, for equilibrium, $\rightarrow = \leftarrow$

$F_r = T \sin \theta = 2Mg \cos \theta \sin \theta$ (2 marks)
 $= 2Mg \left(\frac{3}{5}\right) \left(\frac{4}{5}\right) = \frac{24}{25} Mg$

(d) cont.

Given limiting equilibrium
 $F_r = \mu R = \mu \left(\frac{57}{25}\right) Mg$
 from (c)

So,

$$\frac{24}{25} Mg = \mu \left(\frac{57}{25}\right) Mg$$

$$\Rightarrow \mu = \frac{24}{57}$$

$$= \frac{8}{19} \quad (2 \text{ marks})$$

(1)

(3)

(3)

(4)

$$\cos \theta = \frac{3}{5}$$

$$\Rightarrow \sin \theta = \frac{\sqrt{5^2 - 3^2}}{5}$$

$$= \frac{4}{5}$$

$$R = 3Mg - T \cos \theta$$

$$= 3Mg - 2Mg \left(\frac{3}{5}\right) \left(\frac{3}{5}\right)$$

$$= 3Mg - \frac{18}{25} Mg$$

$$= \frac{57}{25} Mg \quad (1 \text{ mark})$$