



Figure 3

A golf ball is at rest at the point A on horizontal ground.

The ball is hit and initially moves at an angle α to the ground.

The ball first hits the ground at the point B, where $AB = 120\text{m}$, as shown in Figure 3.

The motion of the ball is modelled as that of a particle, moving freely under gravity, whose initial speed is $U\text{ms}^{-1}$.

Using this model,

(a) show that $U^2 \sin \alpha \cos \alpha = 588$

(a) cotd. Resolving vertically (1mark)

$$u = U \sin \alpha$$

$$a = -9.8 \text{ (with +ve } \uparrow)$$

$$s = 0 \text{ when landing}$$

$$s = ut + \frac{1}{2}at^2 \text{ (1mark)}$$

$$0 = U \sin \alpha t + \frac{1}{2}(-9.8)t^2$$

$$0 = U \sin \alpha t - 4.9t^2 \text{ (6)}$$

The ball reaches a maximum height of 10m above the ground.

(b) Show that $U^2 = 1960$

(a) cotd. subst. for $t (= \frac{120}{U \cos \alpha})$

$$0 = U \sin \alpha \left(\frac{120}{U \cos \alpha} \right) - 4.9 \left(\frac{120^2}{U^2 \cos^2 \alpha} \right) \text{ (1mark)}$$

(4)

In a refinement to the model, the effect of air resistance is included.

The motion of the ball, from A to B, is now modelled as that of a particle whose initial speed is $V\text{ms}^{-1}$.

This refined model is used to calculate a value for V

(c) V is greater, since greater initial speed is needed to overcome air resistance and still land 120m away at B.

(c) State which is greater, U or V , giving a reason for your answer.

(d) wind effects / more accurate value for g / dimensions of ball (not particle) / spin of ball etc (1mark)

(d) State one further refinement to the model that would make the model more realistic.

(1)

$$(a) \text{ cotd. } \frac{120 \sin \alpha}{\cos \alpha} - \frac{4.9(14400)}{U^2 \cos^2 \alpha} = 0$$

$$\frac{120 U^2 \sin \alpha \cos \alpha - 70560}{U^2 \cos^2 \alpha} = 0$$

$$= 0 \text{ when numerator} = 0$$

$$\Rightarrow 120 U^2 \sin \alpha \cos \alpha = 70560$$

$$U^2 \sin \alpha \cos \alpha = \frac{70560}{120}$$

$$= 588$$

(1mark)

(b) Resolving vertically (1mark)

$$u = U \sin \alpha$$

$$a = -9.8$$

$$s = 10$$

$$v = 0 \text{ at maximum height}$$

$$v^2 = u^2 + 2as$$

$$0^2 = U^2 \sin^2 \alpha + 2(-9.8)(10)$$

$$U^2 \sin^2 \alpha = 196 \text{ (1mark)}$$

$$\frac{U^2 \sin^2 \alpha}{U^2 \sin \alpha \cos \alpha} = \frac{196}{588} \Rightarrow \tan \alpha = \frac{196}{588} = \frac{1}{3}$$

(1mark)

$$(b) \text{ cotd. } U^2 \sin^2 \alpha = 196$$

$$\Rightarrow U^2 \left(\frac{1}{10} \right) = 196 \Rightarrow U^2 = 1960 \text{ (1mark)}$$

from (a)

$$\Rightarrow \sin \alpha = \frac{1}{\sqrt{1+3}} \Rightarrow \sin^2 \alpha = \frac{1}{10}$$