

In this question, solutions relying on calculator technology are not acceptable.

[In this question, position vectors are given relative to a fixed origin O .]

A particle P is moving in the xy -plane.

At time t seconds, where $t \geq 0$, particle P is moving with acceleration $\mathbf{a} \text{ ms}^{-2}$ where

$$\mathbf{a} = (1 - 3t)\mathbf{i} + (2t^2 - 2t)\mathbf{j}$$

At time $t = 0$

- P passes through the point with position vector $(\mathbf{i} + \mathbf{j})\text{m}$
- P is moving with velocity $(3\mathbf{i} - 2\mathbf{j})\text{ms}^{-1}$

(a) Find the velocity of P at time t seconds, where $t \geq 0$

(3)

(b) Find the position vector of P relative to O , when the acceleration of P is parallel to $(-\mathbf{i} - \mathbf{j})$.

(6)

acceleration is not constant, but varies with t
so we need to use differentiation/integration, rather than 'suvat'

$$(a) \quad \mathbf{a} = \begin{pmatrix} 1-3t \\ 2t^2-2t \end{pmatrix} \quad \mathbf{v} = \int \mathbf{a} \, dt = \begin{pmatrix} t - \frac{3}{2}t^2 + c_1 \\ \frac{2}{3}t^3 - t^2 + c_2 \end{pmatrix} \quad (2 \text{ marks})$$

$$\text{At } t=0, \mathbf{v} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \text{ so } \begin{pmatrix} 0 - 0 + c_1 \\ 0 - 0 + c_2 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \Rightarrow c_1 = 3, c_2 = -2$$

$$\Rightarrow \mathbf{v} = \begin{pmatrix} t - \frac{3}{2}t^2 + 3 \\ \frac{2}{3}t^3 - t^2 - 2 \end{pmatrix} \text{ ms}^{-1} \quad (1 \text{ mark})$$

$$(b) \text{ When acceleration is parallel to } \begin{pmatrix} -1 \\ -1 \end{pmatrix}, \frac{1-3t}{2t^2-2t} = \frac{-1}{-1} \quad (1 \text{ mark})$$

$$\Rightarrow -1 + 3t = -2t^2 + 2t \Rightarrow 2t^2 + t - 1 = 0 \Rightarrow t = \frac{1}{2}, -1$$

Given $t \geq 0$, $t = \frac{1}{2}$ (1 mark)

$$\text{position } \mathbf{r} = \int \mathbf{v} \, dt = \begin{pmatrix} \frac{1}{2}t^2 - \frac{1}{2}t^3 + 3t + k_1 \\ \frac{1}{6}t^4 - \frac{1}{3}t^3 - 2t + k_2 \end{pmatrix} \quad (2 \text{ marks}) \quad \text{At } t=0, \mathbf{r} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\Rightarrow \mathbf{v} = \begin{pmatrix} \frac{1}{2}t^2 - \frac{1}{2}t^3 + 3t + 1 \\ \frac{1}{6}t^4 - \frac{1}{3}t^3 - 2t + 1 \end{pmatrix} \quad \text{When } t = \frac{1}{2}, \mathbf{v} = \begin{pmatrix} \frac{1}{2}(\frac{1}{2})^2 - \frac{1}{2}(\frac{1}{2})^3 + 3(\frac{1}{2}) + 1 \\ \frac{1}{6}(\frac{1}{2})^4 - \frac{1}{3}(\frac{1}{2})^3 - 2(\frac{1}{2}) + 1 \end{pmatrix} = \begin{pmatrix} \frac{41}{16} \\ -\frac{1}{32} \end{pmatrix} \quad (2 \text{ marks})$$