

$$\tan \alpha = \frac{4}{3}$$

$$\Rightarrow \sin \alpha = \frac{4}{\sqrt{3^2 + 4^2}} = \frac{4}{5}$$

$$\Rightarrow \cos \alpha = \frac{3}{\sqrt{3^2 + 4^2}} = \frac{3}{5}$$

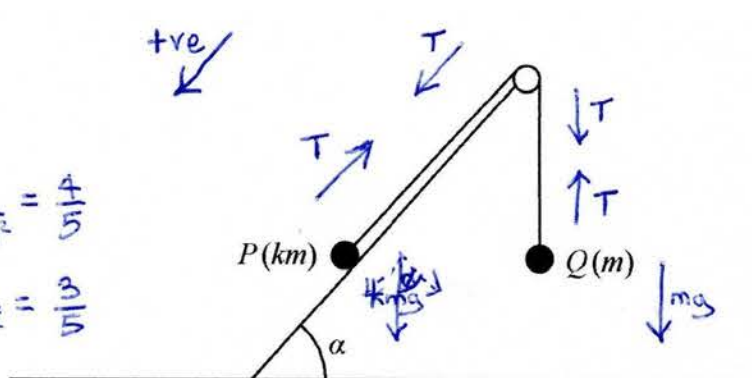


Figure 1

One end of a light inextensible string is attached to a particle  $P$  of mass  $km$ , where  $k > 1.25$

The other end of the string is attached to a particle  $Q$  of mass  $m$ .

The string passes over a small smooth light pulley that is fixed at the top of a plane.

The plane is inclined to the horizontal at an angle  $\alpha$ , where  $\tan \alpha = \frac{4}{3}$

Particle  $P$  is held at rest on the plane and particle  $Q$  hangs at rest with the string taut, as shown in Figure 1.

The part of the string from  $P$  to the pulley lies along a line of greatest slope of the plane.

The two particles and the pulley all lie in the same vertical plane.

The particle  $P$  is released from rest.

In an initial model,

- the plane is modelled as being smooth

- $P$  slides down the plane with acceleration  $\frac{1}{5}g$

Using this model,

(a) write down an equation of motion for  $P$

(a) Resolving parallel to slope,  $F=ma$   
 $R(\swarrow): kmgsin\alpha - T = km(\frac{1}{5}g)$   
 (2 marks)

(b) find the value of  $k$ .

(b) ~~cont~~ Substituting for  $T$ ,  
 $kmgs(\frac{4}{5}) - \frac{6}{5}mg = km(\frac{1}{5}g)$   
 $\frac{4}{5}k - \frac{6}{5} = \frac{1}{5}k \Rightarrow \frac{3}{5}k = \frac{6}{5} \Rightarrow k = 2$  (2 marks)

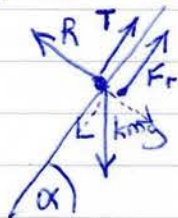
In a second model,

- the plane is modelled as being rough
- the coefficient of friction between  $P$  and the plane is  $\mu$
- $P$  remains at rest but is on the point of slipping down the plane

Using this model,

(c) find, in terms of  $k$ ,  $m$  and  $g$ , the magnitude of the normal reaction exerted by the plane on  $P$ .

(c) We have to also consider the Normal Reaction and Friction now.



$P$  is on the point of slipping down the plane, so friction  $F_r$  is acting up the plane, and is a max ( $=\mu R$ )

Resolving perpendicular to the plane to find  $R$ ,  
 $R(\nwarrow): R = kmgs \cos \alpha$

this is a different model,

so we do not substitute 2 for  $k$

$$R = \frac{3}{5} kmg \quad (2 \text{ marks})$$

(d) find, in terms of  $k$ , the value of  $\mu$ .

(d) We need to calculate the new  $T$

$R(\uparrow)$  for  $Q$ : there is no acceleration, so  $T = mg$  (1 mark)

$R(\swarrow)$  for  $P$ ,  $kmgsin\alpha = F_r + T$  (2 marks)

$$kmgs(\frac{4}{5}) = \mu R + mg \quad (1 \text{ mark})$$

$$\frac{4}{5} kmg = \mu(\frac{3}{5} kmg) + mg$$

$$\frac{4}{5} k = \frac{3}{5} \mu k + 1 \quad (1 \text{ mark})$$

$$4k = 3\mu k + 5 \Rightarrow \mu = \frac{4k-5}{3k} \quad (1 \text{ mark})$$