

3. At time t seconds, where $t \geq 0$, a particle P has velocity $\mathbf{v} \text{ ms}^{-1}$ where

$$\mathbf{v} = (t^2 - 3t + 7)\mathbf{i} + (2t^2 - 3)\mathbf{j}$$

Find

(a) the speed of P at time $t = 0$

(3)

(b) the value of t when P is moving parallel to $(\mathbf{i} + \mathbf{j})$

(2)

(c) the acceleration of P at time t seconds

(2)

(d) the value of t when the direction of the acceleration of P is perpendicular to $\mathbf{i}$

(2)

(a) $\mathbf{v} = \begin{pmatrix} t^2 - 3t + 7 \\ 2t^2 - 3 \end{pmatrix}$ $\mathbf{v}_0 = \begin{pmatrix} 0^2 - 3(0) + 7 \\ 2(0)^2 - 3 \end{pmatrix} = \begin{pmatrix} 7 \\ -3 \end{pmatrix}$

speed = magnitude of velocity = $|\mathbf{v}_0| = \sqrt{7^2 + (-3)^2}$
 $= \sqrt{58} \text{ ms}^{-1}$
 $= 7.62 \text{ ms}^{-1} \text{ (3sf)}$

(b) when parallel to $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, is a multiple of $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ scalar

$$\text{so, } t^2 - 3t + 7 = 2t^2 - 3$$

$$t^2 + 3t - 10 = 0$$

$$(t - 2)(t + 5) = 0$$

$$t = 2, -5$$

but t is > 0 , so $t = 2 \text{ s}$

(c) $\mathbf{a} = \frac{d\mathbf{v}}{dt} = \begin{pmatrix} 2t - 3 \\ 4t \end{pmatrix} \text{ ms}^{-2}$

(d) when acceleration is perpendicular to $\mathbf{i}$, $\mathbf{i}$ component is 0.

$$2t - 3 = 0 \Rightarrow t = \frac{3}{2} \text{ s.}$$