

4. [In this question, $\mathbf{i}$ and $\mathbf{j}$ are horizontal unit vectors and position vectors are given relative to a fixed origin O]

A particle P is moving on a smooth horizontal plane.

The particle has constant acceleration $(2.4\mathbf{i} + \mathbf{j}) \text{ ms}^{-2}$

At time $t = 0$, P passes through the point A .

At time $t = 5 \text{ s}$, P passes through the point B .

The velocity of P as it passes through A is $(-16\mathbf{i} - 3\mathbf{j}) \text{ ms}^{-1}$

(a) Find the speed of P as it passes through B .

(4)

The position vector of A is $(44\mathbf{i} - 10\mathbf{j}) \text{ m}$.

At time $t = T$ seconds, where $T > 5$, P passes through the point C .

The position vector of C is $(4\mathbf{i} + c\mathbf{j}) \text{ m}$.

(b) Find the value of T .

(3)

(c) Find the value of c .

(3)

(a) constant acceleration $\Rightarrow$ can use "suvat"

$$a = \begin{pmatrix} 2.4 \\ 1 \end{pmatrix} \text{ ms}^{-2}$$

$$u = \begin{pmatrix} -16 \\ -3 \end{pmatrix}$$

$$t = 5$$

$$v = ?$$

$$v = u + at = \begin{pmatrix} -16 \\ -3 \end{pmatrix} + \begin{pmatrix} 2.4 \\ 1 \end{pmatrix} 5 = \begin{pmatrix} -16 \\ -3 \end{pmatrix} + \begin{pmatrix} 12 \\ 5 \end{pmatrix} = \begin{pmatrix} -4 \\ 2 \end{pmatrix} \text{ ms}^{-1}$$

$$\text{speed} = \text{magnitude of velocity} = \sqrt{(-4)^2 + 2^2} = \sqrt{20} = 2\sqrt{5} \text{ ms}^{-1}$$

$$\begin{aligned} \text{(b) position, } r_0 &= \begin{pmatrix} 44 \\ -10 \end{pmatrix} & r_t &= \begin{pmatrix} 44 \\ -10 \end{pmatrix} + \begin{pmatrix} -16 \\ -3 \end{pmatrix} t + \frac{1}{2} \begin{pmatrix} 2.4 \\ 1 \end{pmatrix} t^2 \\ & & & \underbrace{\hspace{10em}}_{\Delta (ut + \frac{1}{2}at^2)} \\ & & & = \begin{pmatrix} 1.2t^2 - 16t + 44 \\ 0.5t^2 - 3t - 10 \end{pmatrix} \end{aligned}$$

$$\text{at } C, \begin{pmatrix} 1.2t^2 - 16t + 44 \\ 0.5t^2 - 3t - 10 \end{pmatrix} = \begin{pmatrix} 4 \\ c \end{pmatrix}$$

$$1.2t^2 - 16t + 44 = 4 \Rightarrow 3t^2 - 40t + 100 = 0 \Rightarrow t = 10, 3\frac{1}{3}$$

$$\text{Given } T > 5, T = t = 10 \text{ s.}$$

$$\text{(c) } 0.5(10)^2 - 3(10) - 10 = c \Rightarrow c = 10$$