

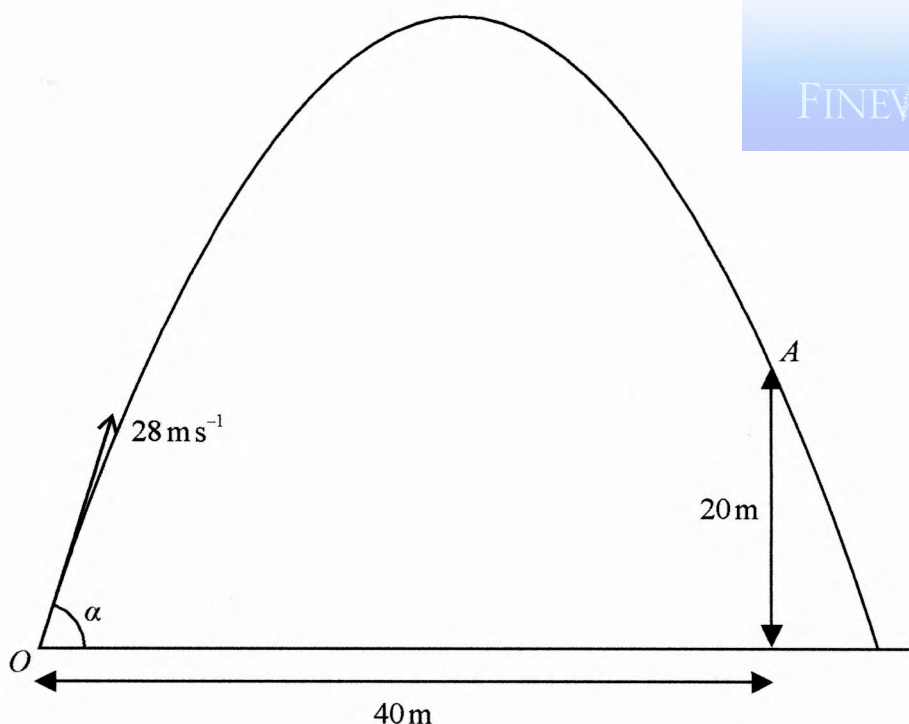


Figure 2

A small ball is projected with speed 28 ms^{-1} from a point O on horizontal ground.

After moving for T seconds, the ball passes through the point A .

The point A is 40 m horizontally and 20 m vertically from the point O , as shown in Figure 2.

The motion of the ball from O to A is modelled as that of a particle moving freely under gravity.

Given that the ball is projected at an angle α to the ground, use the model to

(a) show that $T = \frac{10}{7 \cos \alpha}$

(2)

(a)	Vertically	Horizontally
	$u = 28 \sin \alpha$ $s = 20$ $a = -9.8$ $t = T$	$u = 28 \cos \alpha$ $s = 40$ $a = 0$ $t = T$

$$s = ut + \frac{1}{2}at^2: \quad 20 = 28 \sin \alpha T - 4.9T^2 \quad 40 = 28 \cos \alpha T + 0$$

$$\Rightarrow T = \frac{40}{28 \cos \alpha} = \frac{10}{7 \cos \alpha}$$

(b) show that $\tan^2 \alpha - 4 \tan \alpha + 3 = 0$

(5)

(b) Substituting $T = \frac{10}{7 \cos \alpha}$ for Vertically,

$$20 = 28 \sin \alpha \left(\frac{10}{7 \cos \alpha} \right) - 4.9 \left(\frac{10}{7 \cos \alpha} \right)^2$$

$$20 = 40 \tan \alpha - \frac{490}{49 \cos^2 \alpha}$$

$$20 = 40 \tan \alpha - 10 \sec^2 \alpha$$

$$20 = 40 \tan \alpha - 10(\tan^2 \alpha + 1) \quad (\tan^2 + 1 = \sec^2)$$

$$2 = 4 \tan \alpha - \tan^2 \alpha - 1$$

$$\Rightarrow \tan^2 \alpha - 4 \tan \alpha + 3 = 0$$

(c) find the greatest possible height, in metres, of the ball above the ground as the ball moves from O to A .

(3)

(c) $(\tan \alpha - 1)(\tan \alpha - 3) = 0 \Rightarrow \tan \alpha = 1, 3$
 $\Rightarrow \alpha = 45^\circ, 71.565^\circ$

The larger value of α would project the ball higher, so use $\alpha = 71.565^\circ$

Maximum height is when vertical $v = 0$

$$v^2 = u^2 + 2as$$

$$0 = (28 \sin(71.565^\circ))^2 + 2(-9.8)s \Rightarrow s = 36 \text{ m}$$

The model does not include air resistance.

(d) State one other limitation of the model.

(1)

(d) EG's spin of the ball / dimensions or shape of ball, rather than treating it as a particle / more accurate, or variable, value for g .